Distance measurements using binoculars from ships at sea: accuracy, precision and effects of refraction

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ABSTRACT

The distances to 1,576 targets between 0.3 and 10.4km from two ships were measured using the reticle scale in 25× binoculars during cetacean surveys in the eastern tropical Pacific Ocean. Distances were measured under a range of conditions representing the environmental variability in three years of field surveys. Alternative formulae for calculating distance from optical devices were applied to the reticle measurements and compared to distances measured by radar. Reticles in 25× binoculars provided unbiased measurements to about a third of the way to the horizon, or from 0-4km for the 10.5m platform heights used for the study. Between 4 and 8km (approximately one-third to two-thirds of the distance to the horizon), distances tended to be slightly underestimated, reaching a maximum bias at the most distant targets of 6% for one ship and 16% for the other. Distances beyond about two-thirds of the way to the horizon were not measurable because the angles were too small. The negative bias in measurements of distances from 4-8km was due to refraction of light and other factors. Refraction had less of an effect than expected for a temperature gradient based on a standard atmosphere, suggesting a mean gradient for the eastern tropical Pacific of –0.02°C m^{-1} in the first 10m above the sea surface rather than the standard value of –0.0065°C m^{-1}. Correcting the measurements for refraction improved their accuracy, eliminating the bias for one ship and reducing it for the other. Adjusting for refraction should improve measurements of distance using theodolites or photographic/video imaging as well as measurements using binoculars. An additional regression-based correction suggested that the remaining negative bias for one ship was a complex interaction of Beaufort Sea state, swell height and wind speed. Precision of distance measurements decreased multiplicatively with target distance. Including errors due to bias, the multiplicative standard error was 12%, or a 95% confidence interval from 0.8-1.2km for a target at 1km and from 6.5-9.9km for a target at 8km. Compared with other methods of measuring distance to marine mammals at sea, measurements using binocular reticles are more precise than distances estimated by eye, less precise than distances measured with photographic imaging, and useful over a larger range.

KEYWORDS: TELEMETRY; SURVEY-VESSEL; PACIFIC OCEAN; MODELLING

INTRODUCTION

The distances to marine mammals from a point of observation are fundamental data for estimating abundance using line-transect methods (Buckland *et al.*, 2001), and for some studies of cetacean behaviour (DeNardo *et al.*, 2001; Heckel *et al.*, 2001; Leaper and Gordon, 2001; Frankel and Clark, 2002) and ecology (Fiedler *et al.*, 1998). The distance between a cetacean and an observer can be calculated from the observer's eye-height and the vertical angle between the mammal and a reference line, typically the horizon or shoreline (Lerczak and Hobbs, 1998). This angle is often measured optically using a theodolite, the reticle scale in a binocular or a video/photographic image, and is converted to radial distance using a formula based on spherical geometry (Gordon, 1990; Lerczak and Hobbs, 1998). The measurement can be improved by correcting for refraction (Leaper and Gordon, 2001).

Errors in the estimation of distance in line-transect analyses have been considered by Schweder (1997), Alpízar-Jara *et al.* (1998) and Chen (1998). Underestimation of distance leads to overestimation of abundance and vice versa. Errors in distance measurement can lead to underestimation of abundance even if errors are unbiased (Chen, 1998).

On ship surveys conducted by the Southwest Fisheries Science Center (SWFSC), the angle between a mammal sighting and the horizon is measured using a reticle scale in 25× binoculars (Kinzey and Gerrodette, 2001). This paper examines 1,576 binocular measurements vs radar measurements of distances between 0.3 and 10.4km from two ships for evidence of bias or inaccuracies using reticles. This study compares alternative equations for calculating distance, reports the accuracy and precision obtainable using reticle-based measurements under a range of environmental conditions and introduces: (1) local versus average corrections for the effects of refraction; and (2) additional ship-specific corrections for using reticles in 25× binoculars under field survey conditions. The accuracy and precision of distance measurements obtainable with reticles under field conditions are compared with the accuracy and precision obtainable using naked eye estimates (Schweder, 1997) or using video/photographic images (Gordon, 2001).

METHODS

Converting reticle values to distances

Kinzey and Gerrodette (2001) provide factors to convert reticle values to vertical angles. Lerczak and Hobbs (1998) provide formulae for converting vertical angles to radial distances. Alternative formulae that give equal numerical results for converting angles to distances are given in Gordon (1990), Jaramillo-Legorreta *et al.* (1999) and Buckland *et al.* (2001, p.257).

Two vertical angles are required when binocular reticles are used to measure the distance to a sighting: (1) the angle from a reference line down to the sighting; and (2) an upper angle from the reference line to the horizontal tangent. The first is measured with reticles and the second is calculated from observer height. Both angles, in radians, are summed to calculate distance, D_a, to the sighting in kilometres[1]

[1] The following equation is slightly modified from the form in which it is presented in Gordon (1990) and Lerczak and Hobbs (1998). We thank J.L. Laake, Alaska Fisheries Science Center, for an earlier version of this modified form.

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as follows:

$$D_a = h_e * \sin(\theta + \alpha) - \sqrt{R_E^2 - (h_e * \cos(\theta + \alpha))^2} \qquad (1)$$

where:

θ = angle below the horizon to the sighting, in radians;
α = angle above the horizon to the horizontal tangent = atan $(\sqrt{2R_E h + h^2} / R_E)$, in radians;
h = eye height above sea level, in km;
R_E = radius of earth (= 6,371km);
h_e = $R_E + h$.

θ is referred to as the target angle and α as the above-horizon angle. These angles are also known as 'dip short of the horizon' and 'dip of the visible horizon', respectively (Bowditch, 1995). The distance to the horizon is given by the term $\sqrt{2R_E h + h^2}$ in the definition of α.

Equation 1 can be used to calculate distances from any angle-based device, including theodolites (measuring $\theta + \alpha$ as a single term) or video/photographic images (Gordon, 2001; Leaper and Gordon, 2001). Formulae that produce different numerical results from equation 1 are given in Smith (1982), Buckland *et al.* (1993, p.325), and Bowditch (1995, p.340) (Table 1). Both the Smith (1982) and Buckland *et al.* (1993) formulae use the simplifying assumption of straight-line distance between the sighting and the observation platform rather than accounting for the curvature of the earth (Lerczak and Hobbs, 1998). The Bowditch (1995) formula in Table 1 is an empirically-derived formula used by mariners that accounts for average, worldwide refractive conditions to calculate the dip short of the horizon for an object at known distance.

The 25× binoculars used in this study have no measurable differences in the accuracy of angle measurements among different binoculars (SWFSC 'old style'; Kinzey and Gerrodette, 2001). Each reticle spans 0.0771° (0.00135 radians). The scale is marked to every 0.2 reticles between 0 and 2 reticles and to every half reticle from 2 to 20. The angle between a mammal sighting and the horizon is measured by placing the uppermost reticle line on the horizon and counting the reticles down to the sighting.

Correction for refraction

Equation 1 assumes that light travels in straight lines. It does not account for possible bending due to environmental conditions that can cause refraction (Lerczak and Hobbs, 1998). However, light rays curve when passing obliquely through an atmospheric density gradient (Fleagle and Businger, 1980; Leaper and Gordon, 2001). Light travels faster at lower density and so bends toward higher density when it encounters a gradient. Atmospheric density typically decreases with height (Fleagle and Businger, 1980), which results in a decrease in the perceived angle between a distant object at sea level and the horizontal tangent when the light arrives at an observer. The object is perceived higher relative to the observer than it is based on geometry. This refraction effect is greatest at the horizon, so that although both above-horizon and target angles decrease as a result of refraction, the relative angle between the object and the horizon increases. These combined effects on the target and above-horizon angles result in underestimation of the object's distance when a geometry-based formula such as equation 1 is used.

Equation 1 can be corrected for refraction by using air temperature, air pressure and the vertical gradient in air temperature between target and observer to adjust both the above-horizon and target angles (Leaper and Gordon, 2001). The correction involves calculating the radius of the arc of the refracted ray of light, which is then used to calculate a corrected angle of dip and angle below the horizon. The first empirical term is atmospheric density, A (kg m^{-3}):

$$A = \frac{p\beta}{T} \qquad (2)$$

where:

p = atmospheric pressure in Pa (= 100mb = 100kg m^{-1} s^{-2});
T = air temperature in degrees Kelvin;
β = reciprocal of specific gas constant = 0.00348m^{-2} s^{2} degrees^{-1}.

Atmospheric density is then combined with the temperature gradient to calculate a 'radius of curvature', r, of the refracted ray in meters:

$$\frac{1}{r} = \frac{\varepsilon A}{(1+\varepsilon A)T}\left(\frac{\Delta T}{\Delta h} + g\beta\right) \qquad (3)$$

where:

ε = (refractive index of air — 1)/ air density at sea level = 0.000227m^{3} kg^{-1} for a standard atmosphere at 0°C;
$\frac{\Delta T}{\Delta h}$ = change of temperature with change in height of the light ray = –0.0065^{0}K m^{-1} for a standard atmosphere;
g = gravitational constant = 9.81m s^{-2}.

The $1/r$ value is then used to calculate refraction-corrected horizon and target angles for equation 1, α_c and θ_c, as follows:

$$\alpha_c = \text{atan}\sqrt{2h_m\left(\frac{1}{1000R_E} - \frac{1}{r}\right)} \qquad (4)$$

Table 1

Alternative angle-distance formulae: D_a = distance in kilometres, h = observer eye height in metres above sea level, R_E = radius of earth (= 6,371km), $h_e = h + R_E$, θ = angle from horizon to target in radians, α = angle above horizon to horizontal tangent in radians. The Bowditch (1995) equation was modified from its original form expressing angle in terms of distance by rearranging terms. For the Bowditch formula, observer height (h_f) is in feet, k_1 = 6,076.1 and k_2 = 8,268.

Reference	Formula
Lerczak and Hobbs (1998) (Eq.1)	$D_a = h_e$*sin($\theta+\alpha$) - sqrt[R_E^2 - (h_e*cos($\theta+\alpha$))^2]
Smith (1982)	D_a = 1.852*(h/1852)*tan(atan(89.173/sqrt(h/1852))-θ)
Buckland *et al.* (1993)	D_a = (0.001h)/tan(acos(R_E/(R_E+(0.001h)))+θ)
Bowditch (1995)	D_a =1.852{tan($\theta+\alpha$)k_1k_2 - sqrt[(-tan($\theta+\alpha$)k_1k_2)^2-4$h_fk_1k_2$]}/(2k_1)

and

$$\theta_c \approx \theta + \frac{1000D}{2r} \tag{5}$$

where D = true distance, h_m = observer height in metres above sea surface, and all other terms are defined as for equations 2 and 3 above. These corrected angles can then be used in equation 1 to calculate a corrected distance, D_c, from D_a. Under normal survey conditions, the true distance (D) to the target in equation 5 will be unknown, but it can be initially approximated using D_a from equation 1 to calculate θ_c in equation 5, then substituting θ_c for θ in equation 1 to calculate a new D_a and iteratively repeating this process until D_a converges to D_c. This distance, D_c, is the distance corrected for refraction.

Equation 3 uses the standard temperature gradient of –0.0065°K m^{-1}, which assumes standard atmospheric conditions in the bottom kilometre of the atmosphere (Fleagle and Businger, 1980; Leaper and Gordon, 2001). This is a simplification of the actual situation, where the temperature gradient in the bottom metres of the atmosphere is rarely constant (Fraser and Mach, 1974). The mean gradient along the path the light ray travelled may differ from the standard one, and can be calculated from the observed refraction when true distance to the sighted object is known (Fraser, 1979; Lehn, 1983). Although either positive or negative gradients, indicating increasing or decreasing temperature with height, respectively, are possible near sea level, the typical pattern is decreasing temperature with height as noted above. Equation 3 produces no change in distances calculated from equation 1 at a temperature gradient of approximately –0.034°K m^{-1}, the gradient at which the decreasing temperature with height balances the effect of decreasing pressure to produce a constant density of air (refraction increases as temperature decreases and pressure increases). When air density is constant, no refraction occurs. Refraction will cause underestimates of distance from equation 1 as gradient becomes more positive from –0.034, and overestimates of distance for gradients more negative. As described below, the temperature gradient $\Delta T / \Delta h$ was estimated by fitting equation 3 to the data.

The ε term in equation 3 is based on the refractive index of air of 1.000293 and a density of 1.292kg m^{-3} for a standard atmosphere at 0°C (Lehn, 1983). This term is necessary to weight the measured density by the ratio of the refractive index to refraction calculated at 0°C, and assumes a linear relationship between the index and air density.

Field methods: Distance measurements with reticles and radar

A total of 1,576 measurements of the distances to targets from two ships were made using the reticles in 25× binoculars under a variety of sighting conditions, and paired with radar measurements to the same targets. The reticle measurements were recorded by the regular mammal observers during testing periods on shipboard surveys in the eastern tropical Pacific Ocean in July-December during 1990, 1992 and 1993. 662 of these were made from the NOAA[2] Ship *McArthur* and 914 measurements were made from the NOAA Ship *David Starr Jordan*. Twenty-nine additional measurements made on one day, and 6 that were estimated at less than 0.1 reticles (the normal minimum value used on our surveys) were non-standard or otherwise anomalous and were eliminated from the analysis. These excluded values did not qualitatively affect the results.

Targets ranged between 0.33 and 10.35km from the ships. Within this range, 278 different distances, averaging 0.04km apart, were measured with reticles during the study. The target was generally the waterline of a small boat with a radar target set out for the purpose, but occasionally buoys or other floating objects visible to radar were used. A range of distances between the ship and target was measured during a single testing period by moving either the target (small boat) or the ship (for non-boat targets). To reduce intra- and inter-individual correlations in measurements, observers did not watch the target as it moved to a new position, and did not discuss their measurements with each other. Once the target and ship were in position, three simultaneous measurements with reticles were generally made by different observers together with a single radar measurement to the target. Measurements were made by 24 observers, 16 of whom recorded measurements from both ships and 8 recorded measurements from only one ship. Air temperatures, air pressures, wind speed, sea surface temperature and swell heights associated with the measurements were obtained from the ship deck logs. Ship, Beaufort Sea state and a relative motion code (upswell, downswell, trough) were also recorded.

Binocular heights were fixed, and measured above waterline with a plumb bob while the vessels were at the dock. Observer eye-height above sea level was 10.4m (horizon distance = 11.5km) for the *McArthur*, and 10.7m (horizon = 11.7km) for the *Jordan*. The farthest measurements that could be made using equation 1 at the finest resolution level of 0.1 reticle below the horizon given these platform heights were 7.8 and 8.0km for the *McArthur* and *Jordan*, respectively.

For an object at the farthest measurable distance (0.1 reticle) the target angle, θ (equation 1), is 0.000135 radians. The above-horizon angle (α) for a 10.4m high platform is 0.00181 radians. Although equation 1 is the most geometrically accurate formula for angles of this small magnitude (Buckland *et al.*, 2001), these near-horizon angles are also those for which refraction effects are expected to be greatest (Leaper and Gordon, 2001).

To test accuracy, reticle values were converted to distances using the alternative formulae in Table 1 and compared to distances measured with radar. To evaluate the effects of refraction, the accuracy obtained by correcting reticle measurements for refraction using equations 2-5 was compared to uncorrected equation 1.

Several methods of estimating the terms in the refraction equations were assessed. Refraction corrections using the local air temperature and pressures at the time the reticle measurements were made were compared to those calculated using an average 1/r value. As an alternative to the standard temperature gradient, $\Delta T / \Delta h$ in equation 3 was allowed to be an adjustable variable, with the criterion of minimum logarithmic mean squared error (MSE) between distances from reticles and radar (minimum s_2^2 from equation 8 below) determining the most likely gradient present during each series of measurements taken with the same air temperature and pressure on one day. This produced an estimate of the temperature gradient for each day and an average gradient for the time and region that could be compared to the use of the standard gradient. The results obtained using these various methods for estimating refraction were evaluated on the basis of their data needs and practicality for field studies.

[2] US Department of Commerce, National Atmospheric and Oceanographic Administration.

Calculating precision, bias and accuracy

The variability of 25× measurements of distances corrected and uncorrected for refraction and other factors was assessed in two ways, one that included bias (accuracy) and one that did not (measurement error). Accuracy was calculated using the difference between distances from reticles and radar. Manufacturer specifications indicated that distances from the radar were accurate to within 0.9% or 8m, whichever was greater. Measurement error from reticles was based on the variability of repeated measures to a single target without reference to the true (radar) distance.

Variance of calculated distance D_* increased with true distance D. Log (D_*/D) was approximately normally distributed, indicating that errors were multiplicative rather than additive. A multiplicative standard error for D_* was exp(σ)-1, and an approximate 95% confidence interval was [D/P,DP] where:

$$P = \exp(1.96 * \sigma) \quad (6)$$

and

σ = standard deviation of the logarithm of distance, estimated by s_1 or s_2 as described below.

Three or more reticle measurements were made to 502 separate targets. The standard deviation of measurement error, s_1, was estimated as:

$$s_1 = (\sum_{i=1}^{502} \left[\frac{n \sum_j d_{a,j}^{\ 2} - (\sum_j d_{a,j})^2}{n(n-1)} \right] / 502)^{1/2} \quad (7)$$

where:

n = the number of repeated measurements to a single target (range 3 to 6); and

$d_{a,j}$ = $ln(D_a)$ for the j[th] observation, $j = 1, \ldots, n$.

This calculation of precision indicates the variability of repeated measurements to a target, but not any systematic bias that would cause the mean of those measurements to differ from the true distance. The quantity s_1 will overestimate the accuracy of reticle measurements of distance to the extent that systematic errors result in E(D_a) not equalling D. It represents the maximum precision potentially attainable using unbiased reticle measurements in 25× binoculars at sea, given the variability observed in simultaneous, replicated field measurements.

The second method of estimating σ in equation 6 incorporated bias as well as variability to estimate accuracy. In this method, σ was represented by the root mean squared error between logarithms of distances from reticles and radar, s_2, where:

$$s_2(\text{including bias}) = \sqrt{\frac{\sum (d_{2,k} - d_k)^2}{m}} \quad (8)$$

and

m = total number of paired reticle and radar measurements;

$d_{2,k}$ = logarithm of distance from reticles (d_a or its corrected values, d_c, see below) for the k^{th} measurement, $k = 1, \ldots, m$; and

d_k = logarithm of distance from radar for the k^{th} measurement.

In equation 8, s_2 is calculated using an independent measurement (radar) of the true distance to estimate error in place of a (possibly biased) model estimate. The difference between the two estimates of variability in equations 7 and 8 is an indication of the amount of total variability in reticle measurements that could be due to a biased rather than random component.

In addition to its use calculating confidence intervals in equation 6, the square of s_2 is a measure of the goodness of fit of distances from reticles to radar. Lack of pattern in the residuals of the mean squared errors of logarithms indicated they were a superior measure of this fit compared to mean squared error of unlogged distances, for which residuals increased with distance from the ship.

Correcting distances for bias additional to refraction

Nine variables representing ship motion or other factors potentially influencing measurements using reticles were recorded in addition to the air temperatures and pressures used in the correction for refraction. These included sea surface temperature, year, ship and six factors influencing the motion or average height of the observation platform or target. Sea surface temperature was recorded to test its possible role in refraction. Year and ship effects were examined to see whether additional explanatory factors may have been present but not modelled. These nine variables were coded as: vessel (categorical variable: 1 = *Jordan*, 0 = *McArthur*); Beaufort sea state (continuous: recorded as integers 1-5); ship motion 1 (categorical: 1 = trough, 0 = downswell, 0 = upswell); ship motion 2 (categorical: 1 = upswell, 0 = downswell, 0 = trough); swell height (continuous: in feet); wind speed (continuous: in knots); sea surface temperature (continuous: in °C); year90 (categorical: 1 = 1990, 0 = 1992, 0 = 1993); and year92 (categorical: 0 = 1990, 1 = 1992, 0 = 1993).

The possible affects of these predictors on reticle measurements after correcting for refraction were modelled in two ways using least-squares regression. In each case, predictors were retained or discarded in the final models based on the small-sample Akaiki Information Criterion (AIC_c – Burnham and Anderson, 1998).

In the first set of regressions, the ratio of distance from (refraction-corrected) reticles to radar was the dependent variable predicted by combinations of the nine factors, their squares and pairwise interactions. Thus, the model for the ratio D_c/D of distance from refraction-corrected reticles (D_c) to distance from radar (D), was:

$$D_c/D = \mathbf{bx} + \varepsilon \quad (9)$$

where:

bx = the product of the transposed vector of regression coefficients times the vector of predictor variables selected by AIC_c; and

ε = a normally distributed variable with mean 0 and variance σ_ε^2.

In the second set of regressions, the logarithm of distance from reticles was the dependent variable and the logarithm of distance from radar, its square and pairwise interactions, were additional predictor variables. This model for the logarithm of distance from refraction-corrected reticles (d_c) was:

$$d_c = \mathbf{bx} + \mathbf{b}_r\mathbf{d}_r + \varepsilon \quad (10)$$

where:

bx = the product of the vectors of coefficients and predictor variables (potentially different from those in equation 9), other than factors including radar distance, selected by AIC_c; and

$\mathbf{b}_r\mathbf{d}_r$ = the product of the vectors of coefficients and predictor variables that include the logarithm of radar distance, its square, or interactions.

In the regressions represented by equation 10, true (radar) distance was one of ten possible factors explaining the variability in refraction-corrected distances from reticles. Including true distance as one of the predictors of the reticle value allowed the model to minimise additional variation in reticle measurements due to the other influences once distance was accounted for. In both sets of regressions, the possible combinations of potential predictor variables, pairwise interaction terms and factors squared, were many. Potential variables were added and discarded in stepwise up and stepwise down exploratory fashion, examining hundreds of models, but not all potential combinations were exhaustively explored.

Once a best model (minimum AIC_c) was selected for each of equations 9 and 10, rearrangement of terms to solve for true distance from the initial distance from reticles, independent variables and regression coefficients provided a correction for bias beyond the effects of refraction. This yielded two estimates by equations 9 and 10 of distance from reticle measurements corrected for bias. For equation 9, the model for corrected distance from reticles, D_m, was calculated in a simple rearrangement of the distance from refraction-corrected reticles, D_c, and the associated regression variables and coefficients, as:

$$D_m = D_c/(\mathbf{bx}) \tag{11}$$

with all variables defined as for equation 9.

For equation 10 the final rearrangement involved logarithmic transformations, and so required one additional adjustment to correct for bias in calculating antilogs. This adjustment was based on the property that if the logarithm of x is normally distributed with mean μ and variance σ^2, the expected value of x is $\exp(\mu+\sigma^2/2)$. Thus, the corrected distance from reticles, D_m, based on d_c in equation 10, was:

$$D_{m(k)} = E(X_k) = \exp\left[\frac{bx_k - s^2/2}{\prod_{i=1}^{j} b_{r,i}}\right] \tag{12}$$

for the $k = 1$ to m paired reticle and radar measurements, where:

b = the vector of regression coefficients for the model under consideration, excluding $\mathbf{b}_r$ (equation 10);

x = the vector of (non-distance) explanatory factors for the model;

$b_{r,i}$ = the j regression coefficient(s) for factors including radar distance, its squares and interactions (coefficients for the $\mathbf{d}_r$ in equation 10);

and s^2 was calculated as:

$$s^2 = \frac{\sum_{i=1}^{m}[d_{c(k)} - (bx_k)]^2}{m - df} \tag{13}$$

where:

df = dimension of $\mathbf{b}$ + 1 = the number of coefficients + 1;

k = 1 to m paired reticle and radar measurements (from equation 12) and all other variables are as defined for equation 10.

The calculation of s^2 in equation 13 differs from the squares of s_1 in equation 7 or s_2 in equation 8 in that the value in equation 13 is the deviation from a predicted value based on a model, while the earlier methods of calculating variance did not depend on modelled values. The value in equation 13 was used to correct for bias in calculating antilogs in equation 12, and in calculating an AIC_c for ranking the regression models. The final evaluation of goodness of fit of all the methods of calculating the reticle measurement, D_*, where $D_* = D_a$ or D_c or D_m, was based on minimising s_2, the deviation of D_* from radar, rather than minimising the variance of a model.

RESULTS

Accuracy and precision of distances measured with reticles

Reticle readings fell rapidly with increasing distance to the target (Fig. 1). The reticle values assigned to the targets ranged from 20.5 reticles for the closest to 0.1 reticles for those near the horizon. Thus, in a practical sense, distances could be measured using reticles in 25× binoculars to two-thirds of the way to the horizon, or about 8km. Distances farther than this could not be measured because the angles were too small. Equation 1 provided the best fit of reticles against radar among the formulae tested (Fig. 2). The biases evident in the fits of the Smith (1982; Fig. 2a) and Buckland *et al.* (1993; Fig. 2b) formulae match those discussed from a theoretical perspective in Lerczak and Hobbs (1998). The Bowditch (1995) formula underestimated distances (Fig. 2c).

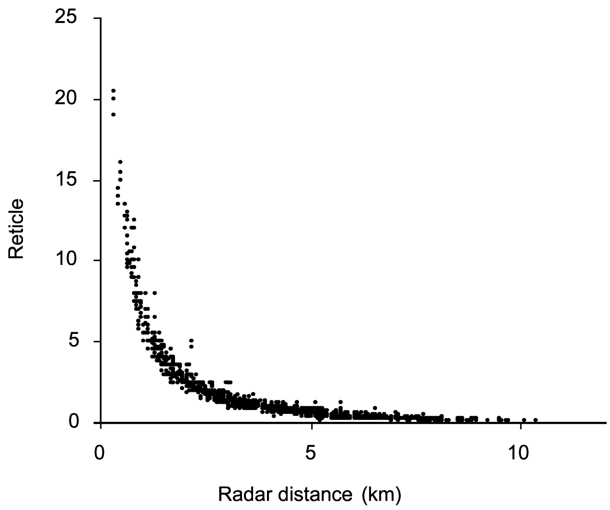

Fig. 1. Distribution of 25× binocular reticle values assigned by observers to targets versus the distances from radar in km to the targets.

Confidence intervals based on estimating σ by equation 8 (accuracy) were wider than those using equation 7 (measurement error). The measurement error (s_1), or

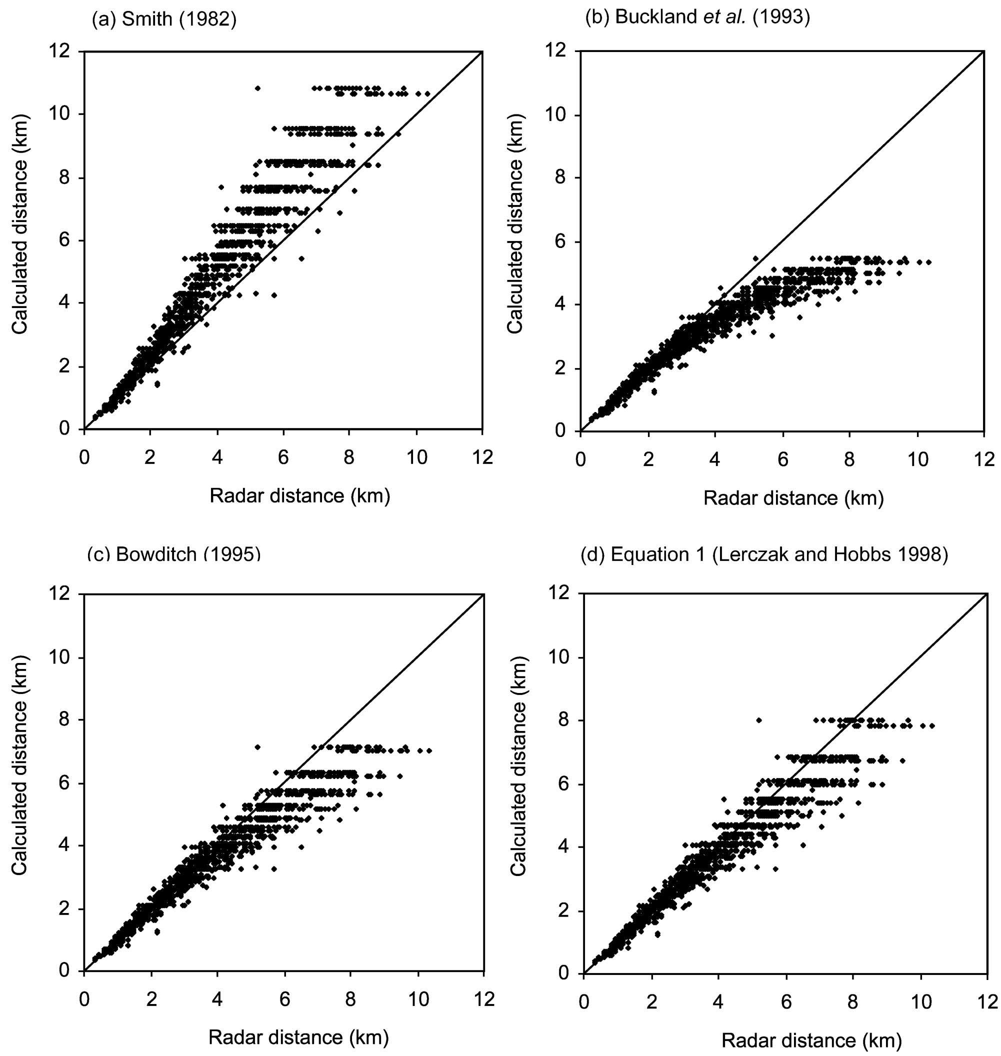

Fig. 2. Distances calculated from reticles using the formulae in Table 1 versus radar. Diagonal lines indicate 1:1 relationship for unbiased reticle measurements of distance. Banding at large distances is due to the discrete values of reticles.

precision of replicate measurements of distance from equation 1 to a single target, was 0.0866 for both ships combined, a multiplicative standard error of 9.0%. This value corresponds to a 95% confidence interval from 0.8-1.2km for a target at 1km, and from 6.8-9.5km for a target at 8km (equation 6). Measurements from the *Jordan* were more precise than those from the *McArthur* (s_1 = 0.0834 vs 0.0909, respectively).

Table 2 lists the mean squared errors, or the variability including bias (calculated as the square of s_2), of the various methods of correcting distances from equation 1 compared to radar. The MSE of the uncorrected distances from equation 1 for the combined dataset was 0.0151 (Method #1 – Table 2), a multiplicative standard error of 13.1%. The uncorrected *Jordan* measurements were closer to radar (MSE = 0.0100) than those from the *McArthur* (MSE = 0.0220).

Although equation 1 produced distances from reticles that agreed well with radar on average, there was a slight tendency to underestimate distances to targets near the horizon (Fig. 2d). For the farthest targets, both ships combined, distance was underestimated by about 10%. A difference between ships was apparent (Figs 3 and 4), with reticle measurements made from the *Jordan* underestimating the distance to targets between 7.5 and 8.5km by 6% on average, or about 0.5km, and measurements from the *McArthur* underestimating these distances by 16%, or about 1.3km. This difference between the ships was unexpected, and suggests either variable refractive effects at the times the measurements were made, or differences between ships other than refraction, as examined below.

Correcting distances from reticles based on refraction

All methods of correcting for refraction improved the mean fit of distances from reticles to distances from radar when the measurements from both ships were combined (reduced the MSEs for the adjusted measurements, Table 2). Using locally measured temperatures and pressures with the standard temperature gradient of –0.0065°C m^{-1} produced an MSE for the combined ships of 0.0125 (Method #7 – Table 2), 83% of the variability for the uncorrected distances. Differences between alternative methods of estimating the terms in the refraction equations were less than the difference between uncorrected equation 1 and the corresponding value from any of the refraction-correction methods.

Air temperatures and pressures during the measurements covered similar ranges on each ship (Table 3). Temperatures were between 15.7 and 31.5°C and pressures were between 100.8 and 101.9 kPa. These values are typical for the eastern tropical Pacific from July to December (da Silva *et al.*, 1994). Air temperatures averaged 25.4°C on the *McArthur* and 25.5°C on the *Jordan*. Air pressure averaged 101.24 kPa on both ships.

Table 2

Accuracy achieved using equation 1 and its corrections (D_*). Results of six methods of calculating distance from reticles are each reported in three ways: once representing the two ships combined, and once for each ship individually. The adjustment terms for air temperature, pressure, and temperature gradient in equations 2-5 were as indicated for each method. Air temperatures and pressures were either those recorded during the measurements ('local') or averaged over the entire study ('average'). Relative reduction in variance is indicated as the ratio between the MSE from the previous column's correction Method (#s 4-18) over the uncorrected MSE (Method #s 1-3) for the same ship(s). Reduced bias is indicated as the ratio of the mean corrected distance from reticles ($D_* = D_a$, D_c, or D_m) over the mean distance from radar (D) approaches 1. NA = not applicable.

Method #	D_*	Distance equations	Air temp/ press	Temp gradient (°C m^{-1})	Ship(s)	MSE (s_2^2)	Relative reduction in variance	Mean D_* / D
1	D_a	Eq. 1	NA	NA	Both	0.0151	NA	0.957
2	"	"	"	"	*Jordan*	0.0100	NA	0.975
3	"	"	"	"	*McArthur*	0.0220	NA	0.932
4	D_c	Eq. 1, 2-5	Average	-0.0065	Both	0.0124	0.82	0.999
5	"	"	"	"	*Jordan*	0.0095	0.94	1.018
6	"	"	"	"	*McArthur*	0.0165	0.75	0.971
7	D_c	Eq. 1, 2-5	Local	-0.0065	Both	0.0125	0.83	0.997
8	"	"	"	"	*Jordan*	0.0095	0.94	0.997
9	"	"	"	"	*McArthur*	0.0165	0.75	0.950
10	D_c	Eq. 1, 2-5	Local	-0.02	Both	0.0132	0.88	0.997
11	"	"	"	"	*Jordan*	0.0093	0.92	0.997
12	"	"	"	"	*McArthur*	0.0188	0.85	0.950
13	D_c	Eq. 1, 2-5	Average	-0.02	Both	0.0132	0.88	0.997
14	"	"	"	"	*Jordan*	0.0092	0.92	0.996
15	"	"	"	"	*McArthur*	0.0187	0.85	0.951
16	D_m	Eq. 1, 2-5, 14	Average	-0.02	Both	0.0119	0.79	0.997
17	"	"	"	"	*Jordan*	0.0096	0.96	0.997
18	"	"	"	"	*McArthur*	0.0152	0.69	0.998

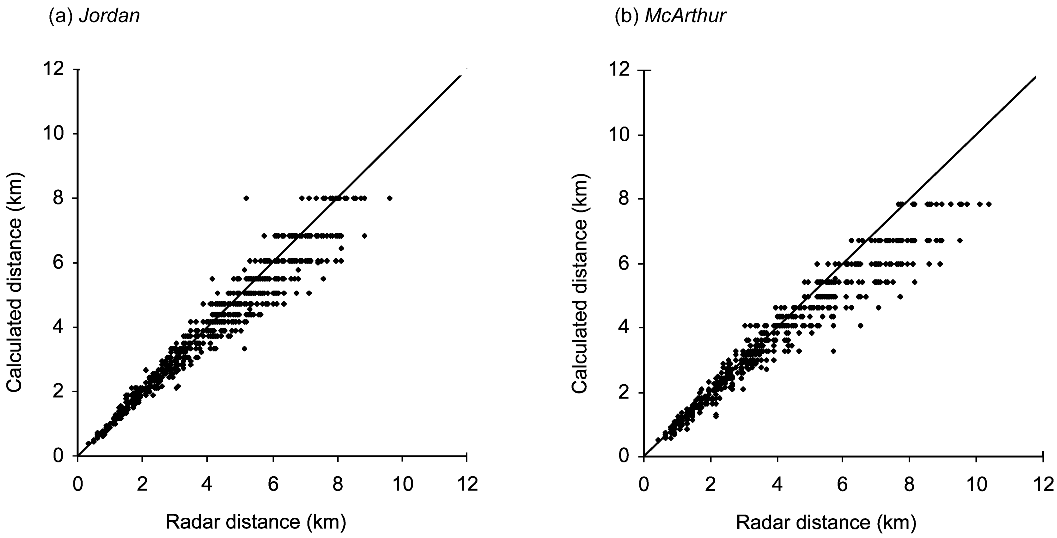

Fig. 3. Differences between ships in the fit of distances calculated from equation 1 (D_a) to radar (D). Uncorrected for refraction or other factors.

Refraction effects were insufficient to account for all of the underestimates of target distances using locally measured air temperatures and pressures with the standard temperature gradient for the *McArthur* measurements. Extreme air temperatures below 0°C, or pressures above 200 kPa (the normal maximum air pressure at sea level worldwide is 104.0 kPa, averaging 101.3 kPa − Fleagle and Businger, 1980), would be required with the standard temperature gradient to produce refractive effects from equations 1-5 sufficient to explain underestimates of the size recorded. The ratio of corrected distance to radar distance using local temperatures and pressures with the standard gradient was 0.950 for the *McArthur* (D_*/D from Method # 9, Table 2), a 5% underestimate on average (note that the bias was nonlinear and so was less than 5% for close targets and more than this for far targets). The *Jordan* ratio of 0.997 was very close to 1, indicating unbiased measurements of distance using reticles for targets at all distances from 0.3-8km from this ship once refraction was accounted for.

Since air temperatures and pressures recorded from the *McArthur* were far from what would be required to produce underestimates of the size observed, the only term left to explain the difference between ships if it was the result of refraction was the temperature gradient, ΔT / Δh. The locally-measured air temperatures and pressures produced 1/r values between 2.38×10^{-8} and 2.67×10^{-8} when combined with the standard temperature gradient. The transformed dip values, α_c, using these ranges were between

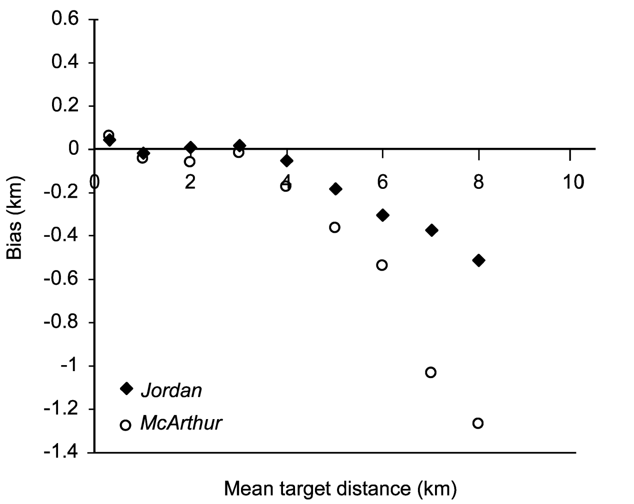

Fig. 4. Bias as a function of distance. Mean differences (bias) between distances measured with reticles (equation 1) and by radar. Measurements are grouped in 1km intervals around each of km 1 to 8, and from 0 to 0.5km, separately for the two ships.

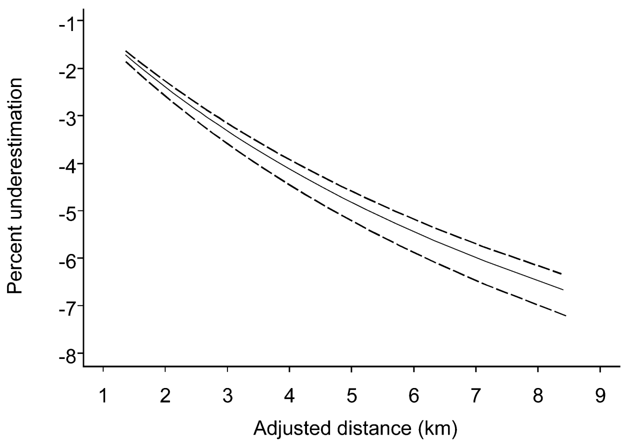

Fig. 5. Sensitivity of the underestimate of distance due to the ranges of air temperatures and pressures measured in this study. The vertical axis represents $1 - D_a/D_c$. The solid line was calculated using average temperatures (25.2°C) and pressures (101.24 kPa). The dashed lines indicate the high and low values around this average due to the range of air temperatures and pressures recorded. All calculations used the standard temperature gradient.

0.00165 and 0.00169 radians. The target angles, θ_c, were increased relative to θ by approximately 10^{-8} radians. The effects of these small angular increases on the calculated distance were most evident for targets near the horizon.

Table 3

Number (N) of paired radar and reticle measurements of distance to targets taken at the given date, water temperature, air temperature, air pressure and ship. Temperature gradients were estimated by fitting $\Delta T/\Delta h$ in equation 3 to minimise the differences in distances from reticles and radar (s_2). Sorted by $\Delta T/\Delta h$ for each ship.

Date yymmdd	N	Water temp °C	Air temp °C	Air press kPa	Fitted $\Delta T/\Delta h$ °C m^{-1}
David Starr Jordan					
900922	55	24.6	21.5	101.29	-0.049
930910	54	20.6	20	101.13	-0.047
930817	53	30	31.5	101.08	-0.046
930731	46	18.1	19	101.74	-0.04
930910	61	20.5	20.2	101.72	-0.039
931017	39	30.5	30	100.87	-0.036
930922	13	19.2	19.5	101.40	-0.031
900825	56	29.5	29.2	101.20	-0.031
900928	63	29.2	27.5	100.80	-0.023
930922	63	19.1	21.5	101.38	-0.021
931017	54	30.2	29.8	100.80	-0.013
930817	60	30.1	30	101.10	-0.009
921009	70	26.7	24.5	101.18	-0.008
920825	114	30.1	29.8	101.20	0.008
920819	23	29.9	28.1	101.18	0.01
930731	90	18.3	20.8	101.75	0.029
McArthur					
901119	42	23.6	26.2	101.38	-0.051
920825	53	31.4	31.1	101.11	-0.014
930728	47	26.3	21	101.50	-0.004
900808	83	26.5	27	101.26	0.001
901119	30	23.6	25	101.29	0.002
900915	39	24.8	24.2	101.31	0.006
901124	102	24.4	26.1	101.16	0.009
920825	53	31	30.9	101.05	0.011
901124	23	24.3	24	101.28	0.012
930821	54	12.5	15.7	101.91	0.013
900821	48	28.3	30.2	100.97	0.024
901119	16	23.6	26.5	101.32	0.034
900821	60	28.3	29	100.84	0.045
900905	12	27.9	26.9	100.99	0.052

Fig. 5 shows the effect this range of $1/r$ values had on correcting distances from reticles for refraction from a 10.4m platform. By 8km, the uncorrected distance from equation 1 varied between about 93% and 94% of the corrected value. The approximately 1% difference attributable to local conditions suggested a standard correction based on average conditions would provide most of the improvement obtainable using local temperatures and pressures (see also Leaper and Gordon, 2001).

Using the average 1/r value of 2.48×10^{-8}, calculated from mean temperature and pressure and the standard temperature gradient (-0.0065°C m^{-1}) from both ships reduced the MSE to 0.0124 for the combined measurements (Method #4 – Table 2), a greater apparent improvement than achieved using local measurements of temperature and pressure. This apparent improvement using averaged rather than locally measured values appeared to be a spurious overcorrection of the underestimate from the *McArthur* due to inaccuracies associated with the use of the standard temperature gradient as discussed below. The *Jordan* ratio of corrected to uncorrected distances indicated a slight overcorrection when average temperatures and pressures were combined with the standard gradient for the refraction adjustment (D_c/D = 101.8%, Method #5 – Table 2). The use of the standard temperature gradient as an average value for the eastern tropical Pacific appears to overestimate the bias due to refraction, as follows.

Fitting the temperature gradient: Average vs standard

Temperature gradients in this study were fitted from observed refractive effects rather than directly measured. Two questions concerning the temperature gradient in equation 3 were: (1) how likely does the standard value of –0.0065°C m^{-1} appear to be a mean value for the gradient in the eastern tropical Pacific given the measurements made during the study; and (2) could different values of this parameter at the times of measurement explain the difference in the bias of reticle measurements observed between the ships?

Sixteen series of measurements were made from the *Jordan* during a single period under the same temperature and pressure (made over 11 different days in 3 years) and 14 such series were made from the *McArthur* (8 days in 3 years). The number of measurements in a series varied from 12 to 114 (Table 3). Estimating the local value of the $\Delta T / \Delta h$ term in equation 3 by allowing it to be an adjustable variable selected to minimise s_2 produced daily temperature/height gradients ranging from –0.05-0.03°C m^{-1} for the *Jordan* and from –0.05-0.05°C m^{-1} for the *McArthur* (Table 3). Although these ranges were similar, 11 of the 14 *McArthur* fittings produced positive temperature gradients, while only 2 of the 16 *Jordan* gradients were positive. The average gradient from the *Jordan* was –0.02°C m^{-1}, while the *McArthur* average was 0.01. The 95% confidence interval for the temperature gradient from the *Jordan* measurements, –0.0107 to –0.0333°C m^{-1}, did not include the standard value of –0.0065. The confidence interval for the gradient from the *McArthur* was much wider, 0.0233 to –0.0334, and included the standard value.

The estimated gradients fit this way would be different for the two ships if refractive conditions were different at the times of measurement, or if non-refractive biases were also present that were inadvertently incorporated into the fittings. In evaluating the use of the standard vs a fitted temperature gradient in parameterising the refraction terms, the possibility of bias other than refraction needs to be considered. If reticle measurements underestimated distances from factors in addition to refraction, fitting the gradient term to these measurements would produce a positive bias in the estimated gradient, overfitting additional error than just the portion due to refraction.

This method of calculating the local temperature gradient would not be feasible under normal survey conditions, when the true distances would not be known and so the local gradient could not be estimated for each sighting. Under normal circumstances an average gradient (either calculated for the region or using the standard value) would need to be used.

There are three lines of evidence against different temperature gradients being the explanation for the differences in bias between the ships. First is the similarity between ships in the environmental variables that it was possible to measure directly (Table 3). Second is the greater variability remaining in the *McArthur* measurements compared to those from the *Jordan* even after allowing gradient to be a free variable. Third is that negative gradients are more common than positive gradients. Together these suggest the *Jordan* mean gradient of –0.02°C m^{-1} is probably a better value for the average rate of change in air temperature in the first 10m above the sea surface in the eastern tropical Pacific in July-December than either the *McArthur* value, or the –0.0065°C m^{-1} value based on a standard atmosphere.

Using a temperature gradient of –0.02°C m^{-1} with the mean measured temperature and pressure resulted in a smaller adjustment to distances from reticles than the standard gradient. Correcting the reticle measurements for refraction using average temperature (25.2°) and pressure (101.24 kPa) and the fitted gradient (–0.02°C m^{-1}) produced a mean ratio of refraction-corrected distance from reticles to radar (D_c/D) of 0.996 for the *Jordan* (Method #14 – Table 2), close to a 1 to 1 relationship on average. Using local measurements of air temperature and pressure with either the standard or fitted gradient improved this *Jordan* ratio slightly, to 0.997 (Table 2). For the *McArthur* the mean ratio after correcting for refraction using average air temperatures and pressures and the (*Jordan*) fitted gradient was 0.951, indicating a continued underestimate of distances from this ship. The distance underestimates from the *McArthur* were apparent for targets farther than 4 or 5 kilometres, while the *Jordan* measurements appeared unbiased after correcting for refraction (Fig. 6).

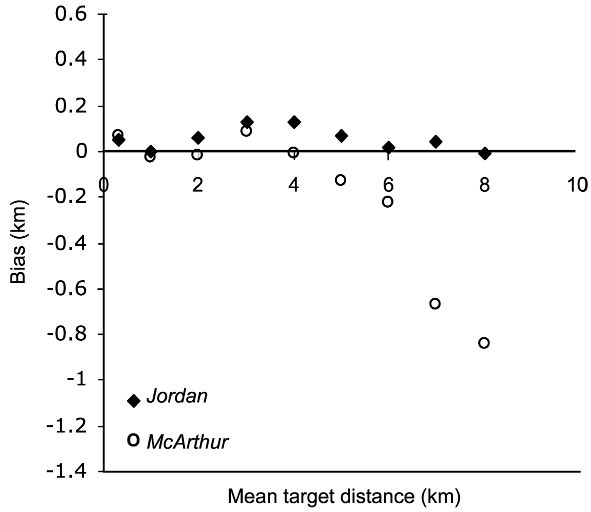

Fig. 6. Bias following correction for refraction. Mean difference by ship between distance from refraction-corrected reticles (D_c) and radar for targets grouped in 1 km intervals. Corrections used average temperature (25.2°C) and pressure (101.24 kPa) and the fitted temperature gradient (–0.02 °C m^{-1}) to adjust the reticle measurements (Methods #14 and #15 in Table 2).

Regression models: Ship, Beaufort, swell and interaction effects

All ratio models based on equation 9 displayed a nonlinear relationship in the errors with target distance. Target distances tended to be overestimated at middle ranges and underestimated at far ranges. This suggested predicting distance from reticles with radar distance as one of the independent factors, rather than assuming the ratio was constant as in the ratio models (i.e. equation 10 rather than equation 9). The best model, selected based on minimum AIC$_c$, for the logarithm of refraction-corrected distance from reticles, d_c, (equation 10) was:

$$d_c = b_0 + b_1(\mathrm{fv})^2 + b_2\mathrm{sv} + b_3\mathrm{f}^2 + b_4\mathrm{fv} + b_5\mathrm{y} + b_6\mathrm{w} + b_7\mathrm{w}^2 + b_8 d + b_9 d^2 \quad (14)$$

where:

f = Beaufort sea state;
v = vessel (1 = *Jordan*, 0 = *McArthur*);
s = swell height in feet;
y = 1 for year 1990, 0 otherwise;
w = wind speed in knots;
d = logarithm of distance, D, from radar;
and b_0 to b_9 are reported in Table 4.

Rearrangement of this model using the quadratic equation to solve for the refraction- and regression-adjusted distance, D_m, produced the correction:

$$\mathrm{Dm} = \exp\left(\frac{-b_6^{-1} - \sqrt{b_6^{-2} - 4b_5^{-1}c}}{2b_5^{-1}}\right) \quad (15)$$

where:

$$c = -\left[\frac{d_c - b_0 + b_1(\mathrm{fv})^2 + b_2\mathrm{wv} + b_3\mathrm{f}^2 + b_4\mathrm{fv} + b_5\mathrm{y} + b_6\mathrm{w} + b_7\mathrm{w}^2 - s^2/2}{b_8 b_9}\right]$$

Table 4

Coefficients for empirical regression model (eq. 14) predicting logarithm of refraction-corrected reticles (d_c) from logarithm of distance (d), or rearranged (eq. 15) to predict model distance (D_m) from refraction-corrected reticles.

Coefficients	Factors	Values
b_0		-0.01625
b_1	(Beaufort*vessel)^2	0.01293
b_2	swell*vessel	0.009993
b_3	Beaufort^2	-0.005584
b_4	Beaufort*vessel	-0.03681
b_5	year90	0.06335
b_6	wind	0.009259
b_7	wind^2	-0.0003381
b_8	d	1.045
b_9	d^2	-0.04616

with s^2 as defined in equation 13. This model had the lowest MSE for the combined ships, 0.0119, of any of the corrections in Table 2, with a multiplicative standard error of 11.5%. The ratio of distances from the best model to radar (D_m/D) was 0.997 over all target distances, and the downward bias remaining in the refraction-only corrected *McArthur* measurements (Fig. 6) was removed (Fig. 7).

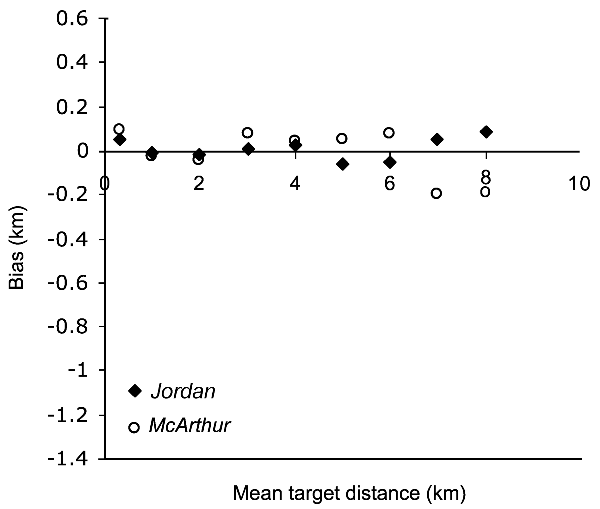

Fig. 7. Bias following correction by regression modelling (equation 15). Mean difference by ship between distance from regression-corrected reticles (D_m) and radar for targets grouped in 1km intervals. Reticles were corrected for refraction using average temperatures and pressures and the fitted gradient before modelling additional, ship-specific factors (equation 14) and solving for empirically-corrected distance (equation 15).

The six factors in equation 14 interact in a complex, nonlinear fashion to produce reticle values from the true distance combined with three factors affecting ship motion (Beaufort, swell height and wind speed), and two categorical variables, one representing ship, and the other a year effect in 1990. The inclusion of the latter two variables indicated that a complete explanation of the difference in measurement bias either included more factors or had a different structure than the models considered in this study. The empirical model distinguished between some of the important and unimportant factors and was useful in a predictive sense. It indicated that relative to the *Jordan*, the *McArthur* bias increased with Beaufort, swell height and wind speed. Water temperatures and the ship's course relative to the swell direction (categorical motion codes ship motion 1 and ship motion 2) were not important in reducing the variance of the estimates.

Table 5 and Fig. 8 summarise the precision and accuracy for the three methods of calculating distance considered in this paper. Reticle measurements from both ships were grouped into eight sets or blocks. Each block was composed of all radar measurements within a 1km interval, centred on integer distances from 1-8km. The 95% confidence interval (equations 6 and 8) and mean bias (D^*/D) of distance from reticles against radar was calculated for each of the blocks. The product of each confidence limit and mean bias illustrates the improvements obtained using the corrections.

The results from uncorrected equation 1 included all the sources of bias and variability that were present during the tests. These had little effect on the precision and accuracy for targets closer than about 4km. Beyond 4km, the confidence intervals widened and there was a tendency to underestimate distance. The refraction and empirical regression corrections in Fig. 8 show the improvements achieved in measurements of radial distance using the methods discussed in this study. The 95% confidence intervals improved both in terms of precision and reduced bias.

DISCUSSION

Accuracy and precision

This study identified the accuracy with which distances can be measured from ships using the reticles in 25× binoculars, provided empirical support for the theoretically derived equation 1 over alternative equations, and explored the effect of refraction on distance measurements in the eastern tropical Pacific Ocean in July-December. It also quantified small differences between two ships in the precision and bias of these measurements.

In light of these findings, the first questions a researcher using angle-based measurements of distance should ask are (1) how far from the sighting platform will the sightings be and (2) what level of accuracy is required to meet the research objectives? The underestimate of distances from uncorrected reticle measurements effectively disappeared for objects closer than about 4km, or a third of the way to the horizon, for both ships in the study. For measurements closer than this the practical effect of the corrections would be negligible, and distances computed with equation 1 should suffice. This is true of most of the radial sighting distances obtained during SWFSC field surveys. Researchers measuring distances nearer the horizon with optical devices who require accuracy better than the 6-16% mean underestimate for targets at 0.1 reticles may want to consider the types of corrections discussed here, however.

The accuracy and precision of distances measured at sea for biological studies has been assessed in two other studies at shorter distances (0-2km) than tested in this study (0-8km). Using video and still cameras, Gordon (2001) reported accuracy as absolute mean percentage error from 2.6-6.4%. This included error due to the independent measurement of range by non-differential GPS or laser range-finding binoculars. Percentage error over a similar range of distances in this study was 8.2%. Video and photographic methods are currently limited in range by image quality (Leaper and Gordon, 2001). Distance measurements made by eye over a range of 0-2km had a negative bias of 9% at close distances and less bias at 2km (Schweder, 1997). The multiplicative standard error of

Table 5

Precision and accuracy at eight distances using three methods of computing distance. Bias (D_*/D), multiplicative standard error ($SE_* = (\exp(s_{2*})-1)$), lower (L_*) and upper (U_*) 95% confidence limits are shown at each distance for each method. The three methods, indicated by subscripts, are: a = without correction for refraction (eq. 1), c = with correction for refraction (eqs. 2-5), and m = with empirical regression correction for other factors (eq. 15). n = number of measurements within each distance category.

Midpoint of radar distances (D)	n	Uncorrected reticles (a)				Refraction-corrected (c)				Regression-corrected (m)			
		D_a/D	SE_a	L_a	U_a	D_c/D	SE_c	L_c	U_c	D_m/D	SE_m	L_m	U_m
1	148	1.04	12.8	0.8	1.3	1.05	12.7	0.8	1.3	1.05	11.6	0.8	1.3
2	209	0.99	12.0	1.6	2.5	1.00	12.0	1.6	2.5	0.98	12.1	1.6	2.5
3	245	1.00	9.8	2.5	3.6	1.02	10.0	2.5	3.7	1.01	9.9	2.5	3.6
4	249	0.98	10.3	3.2	4.7	1.00	10.0	3.3	4.8	1.01	10.1	3.3	4.9
5	234	0.96	12.0	3.8	6.0	0.98	11.1	4.0	6.0	1.00	11.2	4.1	6.2
6	205	0.93	13.0	4.4	7.1	0.96	11.6	4.6	7.1	1.00	11.0	4.9	7.4
7	136	0.91	16.0	4.8	8.5	0.94	14.1	5.1	8.5	0.99	11.5	5.6	8.6
8	88	0.87	18.8	5.0	9.8	0.90	16.4	5.4	9.7	0.98	12.3	6.2	9.8
Means		0.96	13.1			0.98	12.2			1.00	11.2		

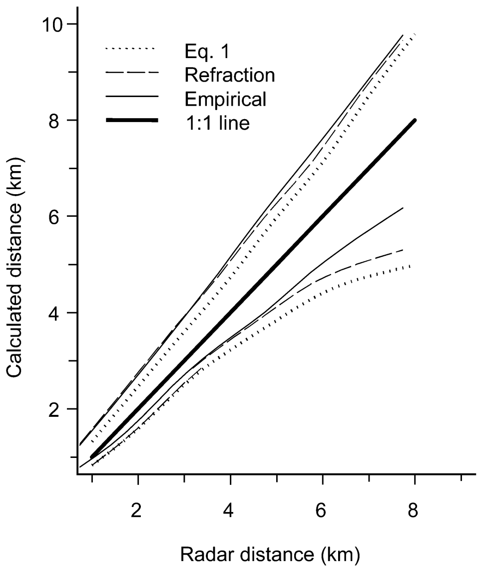

Fig. 8. 95% confidence intervals for measurements of distance using reticles, from Table 5. The 1:1 line indicates measurements without variance or bias.

distances estimated by eye (36%) was three times the value for distances measured by binocular reticles in this study (12%).

Refraction

Effects of refraction in measuring distance were responsible for small but measurable underestimates of distances beyond about one third the distance to the horizon. Corrections to account for refraction removed the downward bias from one ship but not the other. The corrections for refraction incorporated either the locally-measured or mean air temperature, air pressure and temperature gradient into a single term, $1/r$, which was then used to modify the two angles associated with each sighting. In the typical circumstance of decreasing air density with height, both angles were somewhat reduced relative to their geometrically expected values in the absence of refraction.

Refraction in the study region appeared to be less than predicted from the temperature gradient based on a standard atmosphere. The standard gradient is a worldwide average and includes polar, terrestrial and other areas where the average rate of change in temperature with height above the earth's surface might differ from the study region. The mean gradient calculated by empirically fitting the $\Delta T / \Delta h$ term in this study was more negative than the standard one, indicating a stronger decrease in temperature with height in the first 10m above sea surface in the study area than the standard value. This stronger gradient produced less refraction by reducing the change in air density with height, relative to the standard gradient. Refraction accounted for about half of the underestimate of distance (approximately 5%) for the farthest measurements in this study.

Refraction effects would be greater than in the eastern tropical Pacific in regions of colder air temperatures, higher pressures and/or a less negative temperature gradient. Leaper and Gordon (2001) calculate an underestimate of about 10% for measurements made to 12km at air temperature 0°C, pressure 100.0 kPa, and temperature gradient of 0°C m^{-1}. A positive temperature gradient above the sea surface would cause even more of an underestimate.

Extreme gradients of air temperature produced numerically undefined results in equations 2-5. Gradients between about –0.519 and 0.027°C m^{-1} produced defined solutions with the average temperature and pressure recorded in the eastern tropical Pacific. For more extreme combinations of air temperature, pressure and air temperature gradient, the refraction solutions became unstable, reversing direction with changing gradient as the limits were approached before becoming undefined. For instance, the solution was undefined at a gradient of –0.520, reduced the distance to 77% of its uncorrected value at a gradient of –0.519, had no affect on distance when gradient was approximately –0.514, and increased the distance to 148% of the uncorrected value when the gradient was –0.423. From –0.423 to –0.034 the effect again decreased to zero and then increased as the gradient became less negative than –0.033. Analogous behaviour in the physical system may correspond to mirage or other visual distortions (Fraser and Mach, 1974; Fleagle and Businger, 1980).

The maximum underestimate produced by the refraction equations, other than near the limits of the range of gradients that produce extreme and unstable numerical results as described above, was about 13%. For example, distances at 0.1 reticle from a 10.4m platform were reduced by this much at air temperature 0°C, pressure 1012 hPa and a $\Delta T / \Delta h$ of +0.01°C m^{-1}. Changing any of the three environmental terms in either direction caused less of an underestimate.

The correction for refraction warrants consideration anytime distance measurements are to be made near the horizon with an angle-based optical device if mean accuracy better than about 5-13% (the range of adjustments obtainable from equations 2-5 using realistic temperatures, pressures and gradients) is desired. Refraction will be greatest in cold air temperatures with a positive gradient near the surface. Researchers conducting studies under such conditions could use equations 2-5 with the standard gradient to estimate the likely magnitude of refraction expected for the region. If this suggests inaccuracies due to refraction larger than acceptable for the research objectives, field measurements to estimate local temperature gradients may be warranted. Optical measurements of distance could be calibrated against targets of known distance. For a stationary platform such as a theodolite on a clifftop, local refractive conditions might be checked regularly against a buoy or similar target. For shipboard measurements, a calibration system using radar or similar range-finding equipment would likely need to be used.

Swell, Beaufort, wind and ship effects

The regression modelling was to clarify and explain any additional features affecting reticle measurements from ships at sea beyond the effects of refraction. Reticle measurements from the *McArthur* underestimated distances more than expected from refraction alone. This was the largest portion of the underestimate from both ships combined, both before and after refraction had been accounted for. The regression model suggested that biasing factors in the underestimate of distances remaining after correcting for refraction were a complex interaction among Beaufort Sea state, swell height, wind speed and ship. There was also a small year effect, with 1990 differing from 1992 and 1993. None of these effects were large individually. The average difference between the distance predicted by the empirical regression model and that calculated from reticles corrected for refraction only was 0.15km. The average difference due to the year effect was less than 0.001km.

The presence of both year and ship effects in the empirical model is an indication that the physical factors included in the modelling and/or the model structures considered did not completely explain the underestimate of far distances from the combined ships. The year effect was small, but the difference between the ships that appeared as three interaction terms in the empirical model suggests that the results of the regression modelling should not be automatically applied to new, uncalibrated platforms. New platforms would require additional field measurements to targets at far distances to determine whether bias beyond the effects of refraction is present.

The *McArthur* was the more active of the two ships under similar sea conditions. If differences in ship responsiveness resulted in observers reading reticles differently as ship motion increased, for instance tending to read more at the top of a swell on the *McArthur* than on the *Jordan*, the effective height on the *McArthur* would increase and the results observed in the data would be obtained. Gordon (2001) discusses the opposite effect of ship rolling or heeling, which will result in distances being overestimated. This heeling effect was not apparent in the data used here, however.

As a check on the 'effective height' of the ships, platform height was used as an adjustable variable minimizing s_2 (equation 8) for each ship, using average temperatures and pressures and the fitted gradient. For the *Jordan* this produced a fitted height of 10.8m, close to the measured 10.7. For the *McArthur*, a minimum s_2 of 0.1256 was obtained at a height of 11.2m, compared to the s_2 of 0.1369 for these values with the measured height of 10.4m. Using the standard gradient, the fitted heights were 10.5m and 10.9m for the *Jordan* and *McArthur*, respectively. The reason for the 0.5 to 0.8m difference between effective height and measured height on the *McArthur* is unclear, but could be due to differences in ship responsiveness to sea state or some other unmeasured variable. Even with the empirical height adjustment, measurements on the *McArthur* were more variable than those on the *Jordan*.

Barlow *et al.* (2001) examined factors affecting the perpendicular sighting distances (= radial distance × sine of the horizontal angle from the ship's trackline) to marine mammals from the two ships used in this study. Their results were interpreted in terms of whether or not particular species were seen with distance from the trackline under different sighting conditions. The effect of sighting conditions on perceived (radial) sighting distances in this study even after a target was located suggests another possible avenue by which perpendicular distances could be influenced. This effect would be the same regardless of species, but would differ by ship.

Barlow *et al.* (2001) found both swell and Beaufort sea state to be important factors affecting the perpendicular sighting distances of marine mammal sightings from the two ships, but did not find a ship effect. As sightings are made closer to the ship's trackline, differences in radial distance become smaller on an absolute scale relative to perpendicular distance (sightings on the trackline are all 0km perpendicular, regardless of radial distance). This would decrease the effect of differences in radial distance.

ACKNOWLEDGEMENTS

We thank the observers who made the reticle measurements, and the officers and crew of the NOAA Ships *McArthur* and *David Starr Jordan* for providing radar distances to the targets, conducting the small boat operations, and overall professional support. Daniel Fink produced and edited the initial database from the field measurements. We would also like to thank Russell Leaper and Jonathan Gordon for discussion of the refraction calculations. The manuscript was improved following suggestions by Karin Forney, Rod Hobbs, André Punt and an anonymous reviewer. The analysis portion of this study was made possible through funding by the International Dolphin Conservation Program Act to the Southwest Fisheries Science Center.

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Date received: April 2003.
Date accepted: July 2003.