A framework for evaluating *Strike Limit Algorithms* for populations reduced to small numbers

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ABSTRACT

A generic framework is outlined within which operating models for populations reduced to low numbers but still subject to exploitation ('type 3 fisheries') can be developed. This framework is founded on an individual-based operating model that includes temporally correlated environmental variation in births and survival as well as the possibility of occasional catastrophic reductions in survival. Methods are developed to specify the value of the parameter that determines the productivity of the resource from that for *MSYR*, to enable simulation trials based on this framework to be parameterised in terms of *MSYR*. Three potential candidate *Strike Limit Algorithms* are evaluated using 14 'generic' simulation trials that capture a range of factors pertinent to type 3 fishery situations. The 'Maximum-likelihood-like' *SLA* developed for use in the management of the Bering-Chukchi-Beaufort Seas stock of bowhead whales performs adequately for many of these 14 trials, but not all. In contrast, a variant of the 'PBR approach' is shown to perform adequately in terms of achieving conservation objectives for all of the trials. The information needed to specify trials for actual type 3 fishery situations is outlined.

KEYWORDS: ABORGINAL WHALING; INDIVIDUAL-BASED MODEL; STRIKE LIMIT ALGORITHM; MODELLING

INTRODUCTION

The Scientific Committee of the International Whaling Commission (IWC) has, since 1996, been developing candidate *Strike Limit Algorithms*[1] (*SLA*s) for the management of aboriginal subsistence whaling. The approach adopted by the Scientific Committee to contrast the performances of candidate *SLA*s is Monte Carlo simulation. This approach was used to develop the Revised Management Procedure (RMP) for commercial whaling (Kirkwood, 1997) and has also been applied to develop management procedures for several fish stocks (see, for example, the review by Butterworth and Punt (1999)).

IWC (1997) determined that (initial) priority should be given to developing *SLA*s for two types of fishery:

(1) 'type 1 fishery': a case where there is relatively little available information and stock identity problems and where the Scientific Committee has had considerable problems in providing management advice;
(2) 'type 2 fishery': a case where there is a relatively large amount of information and satisfactory management advice can be provided.

Subsequently (IWC, 1998), a third fishery type was identified. This type is characterised by a high extent of depletion to a small (of the order of 300 animals) total population size. Progress towards the selection of an *SLA* for one of the type 2 fisheries (the Bering-Chukchi-Beaufort Seas stock of bowhead whales) has been completed (IWC, 2003) and simulation trials have been designed for one example of a type 1 fishery (IWC, 1998). In contrast, relatively little progress has been made towards selecting *SLA*s for type 3 fisheries (although see Breiwick and DeMaster, 1999).

What distinguishes type 3 situations (and consequently may necessitate a qualitatively different approach to their management and the associated evaluations) is the relatively

much greater influence on their dynamics of 'demographic stochasticity' (annual variation in the number of births and deaths as a consequence of the fact that the population is comprised of individual animals), environmental stochasticity (annual variation in birth and survival rates common to groups of individuals) and catastrophic events (infrequent events, but ones that have a large impact on groups of individuals)[2] (Shaffer, 1981; Lande, 1993).

The objectives of management for aboriginal whaling have been set by the IWC as follows:

(1) ensure risks of extinction not seriously increased (highest priority);
(2) enable harvests in perpetuity appropriate to cultural and nutritional requirements (implicit in this is the concept of catch stability);
(3) maintain stocks at highest net recruitment level and if below that ensure they move towards it.

Arguably, the most important step in the process of evaluating candidate *SLA*s is the development of an appropriate set of simulation trials. This set reflects different assumptions for an operating model[3]. The trials for fishery types 1 and 2 have been largely based on density-dependent age- and sex-structured operating models, characterised by the BALEEN II model underlying the HITTER-FITTER package (de la Mare, 1989; Punt, 1999). Although some of the operating models for the type 1 and 2 fisheries allow for demographic stochasticity and environmental stochasticity in births (e.g. IWC, 2000), they are 'lumped' (in the sense that the behaviour of individuals is ignored and all animals of a given age are assumed to be interchangeable). It is unclear, however, whether such lumped operating models are appropriate for evaluating *Strike Limit Algorithms* for type 3

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[1] An algorithm that determines the number of allowable strikes from a population subject to an aboriginal harvest.

[2] The impact of catastrophic events may be more severe for small than for large populations because the population may be reduced to levels at which demographic stochasticity may render the resource extinct.
[3] The model that represents the true situation for the simulation trials.

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fisheries because the behaviour of individuals may play a large role in the dynamics of the population when the population size is small.

This paper therefore develops an operating model that is individual-based. The behaviour of this model as the values for its parameters are changed is examined and this behaviour is contrasted qualitatively with that of a lumped operating model. The performances of three potential candidate *SLA*s (including one based on the PBR approach (Wade, 1998) and another based on the 'Maximum-likelihood-like' *SLA* developed by Punt (2001)) are then contrasted by using a set of illustrative fishery type 3 trials. Finally, the further work needed to apply the framework developed in this paper to actual type 3 fisheries (e.g. Cook Inlet white whales (*Delphinapterus leucus*), Baffin Bay bowhead whales (*Balaena mysticetus*) and the humpback whale (*Megaptera novaeangliae*) feeding aggregation off West Greenland) is identified.

METHODS

The stochastic individual-based model

The model described below considers the population at the level of the individual. The information available for each animal includes: sex; age; whether recruited to the fishery or not; whether mature or not (females only - animals aged 60 and older are assumed not to calve); the animal's mother and the year in which the animal (if female and mature) last calved. The following steps occur during each year y to project the model forward.

(1) Generate the survival rate for year y for animals of sex s and age a, $S^s_{y,a}$ (the survival probability can be temporally correlated or the population can be subject to catastrophic mortality events - see Appendix A for details).

(2) Compute the number of 1+, mature and recruited animals at the start of year y.

(3) Compute the probability of a mature animal giving birth, f_y (see Appendix B).

(4) Determine, for each mature female that did not give birth the previous year or whose calf died during its first year, whether it gives birth at the start of year y by conducting a Bernoulli trial with probability f_y.

(5) Determine the sex of each new calf by conducting a Bernoulli trial with probability 0.5 for each calf.

(6) Remove the catch. If fishing involves a fixed harvest rate, a Bernoulli trial with a probability of success equal to the harvest rate is conducted for each recruited animal to determine whether it is caught or not. If the numbers caught (by sex) rather than the fishing mortality rate is specified, then this catch is removed by selecting animals at random (and without replacement) from the set of recruited animals (i.e. multinomially). If a female that calved during year y is harvested during year y, its calf is assumed to die.

(7) Determine, for each animal that survives the harvest, whether it survives natural mortality by conducting a Bernoulli trial with probability $S^s_{y,a}$. If a female that calved during year y dies due to natural causes during year y, its calf is assumed to die as well.

(8) Determine, for each unrecruited animal, whether it recruits at the end of the year by conducting a Bernoulli trial with probability of success of

$$\delta_a = (R_a - R_{a-1})/(1 - R_{a-1})$$

where R_a) is the probability that an animal of age a is recruited[4].

(9) Determine, for each immature female, whether it matures at the end of the year by conducting a Bernoulli trial with probability $(M_a - M_{a-1})/(1 - M_{a-1})$ where M_a is the probability that an animal of age a is mature.

(10) Increment the age of each animal.

Specifying the values for the parameters of the model

The values for the biological parameters of the model have been set to those 'typical' for a large baleen whale (based on the suggestions by Breiwick and DeMaster (1999) - Table 1). For consistency with previous *SLA* evaluations (IWC, 2002), all animals are assumed to recruit to the fishery at age 1, i.e. $R_0 = 0$ and $R_1 = 1$ for ages $a = 1$ and older. The value of the parameter z has been assumed to be 2.39. This choice for z corresponds to *Maximum Sustainable Yield Level* $(MSYL) = 0.6$ for a model in which the recruitment and maturity ogives are identical and density-dependence acts on the mature (or recruited) component of the population.

The remaining parameters of the model determine the current (1+) population size, the current depletion (current 1+ abundance relative to its average pre-exploitation level), $MSYR_{1+}$[5], the impact of catastrophic events, whether survival is subject to temporally correlated fluctuations and whether the probability of birth is subject to temporally correlated fluctuations. Fourteen scenarios (Table 2) examine the sensitivity of the results to a range of hypotheses about these factors. Current (1+) abundance is low (300) for all but two cases and the base-case value for *MSYR* is 2.5%. The majority of the trials ignore catastrophic events and temporally correlated birth and survival rates (equivalent to the SD (Demographic Stochasicity) assumptions underlying the trials developed for the Bering-Chukchi-Beaufort Seas stock of bowhead whales - IWC, 2002).

The specifications related to productivity in Table 2 are expressed in terms of the *MSY* rate, *MSYR*. However, the model parameter determining productivity is A and not *MSYR* (see Equation B.1). Unlike the deterministic case

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[4] This equation arises from the relationship between δ_a and R_a: $R_a+1 = R_a + \delta_a(1 - R_a)$, i.e. the number of animals recruiting at age $a+1$ is δ_{a+1} multiplied by the number of animals of age $a+1$ that have not yet recruited.

[5] *MSYR* is the ratio of *MSY* to the population size at which *MSY* is achieved.

Table 1

Expected survival probabilities and the probability of being mature as a function of age.

Age	0	1	2	3	4	5	6	7	8	9	10+
Survival, S_a	0.75	0.75	0.8	0.85	0.9	0.92	0.94	0.95	0.96	0.97	0.98
Maturity, M_a	0	0	0.25	0.375	0.5	0.5	0.625	0.75	1	1	1

Table 2
Specifications of the 14 simulation trials.

Description	Trial no.	Current population size	Current depletion	Catastrophic events	CV, ρ of env. variation in survival	$CV, \tilde{\rho}$ of env. variation in births	$MSYR_{1+}$ (%)
Base-case	1a	300	0.05	No	0	0	2.5
	1b	300	0.1	No	0	0	2.5
Low $MSYR_{1+}$	2b	300	0.05	No	0	0	1
	2b	300	0.1	No	0	0	1
High $MSYR_{1+}$	3a	300	0.05	No	0	0	4
	3b	300	0.1	No	0	0	4
Large population size	4a	3,000	0.05	No	0	0	2.5
	4b	3,000	0.1	No	0	0	2.5
With catastrophic events	5a	300	0.05	Yes*	0	0	2.5
	5b	300	0.1	Yes*	0	0	2.5
With env. variation in survival	6a	300	0.05	No	0.025, 0.71	0	2.5
	6b	300	0.1	No	0.025, 0.71	0	2.5
With env. variation in births	7a	300	0.05	No	0	0.5, 0.71	2.5
	7b	300	0.1	No	0	0.5, 0.71	2.5

* A catastrophic event involves an 80% reduction in population size The probability of a catastrophic event is 2% each year.

(Punt, 1996), the expected sustainable catch is not related analytically to *A* and *MSYR*. Instead, it is necessary to apply a numerical approach to solve for *A* given *MSYR* and the values for the remaining parameters of the model (Appendix C). This approach is similar to that used in New Zealand to define *MSY* for fish species with highly variable recruitment (Francis, 1992). The initial conditions for the population projections correspond to a resource at its (deterministic) pre-exploitation level. Two levels for the depletion at the start of the simulations (0.05 and 0.1) are considered. Both of these correspond to a resource well below conventional target levels.

Each simulation trial involves projecting the population from pre-exploitation equilibrium for 200 years without catches (so that the age-structure at the start of the first year in which historical catches are taken is not in equilibrium because of the impact of random variation in births and deaths), and then removing 100 years of historical catches (the catches are removed under the assumption of a constant intended harvest rate over the 100 years – the actual harvest rate (the ratio of the number harvested to the 1+ population size) will differ from the intended harvest rate given the Bernoulli process used to decide whether an animal is harvested or not). The 250 simulations that constitute a simulation trial are each constructed by selecting the random variates that determine stochasticity in birth and death rates and in catastrophic events and then varying the harvest rate over the historical period so that the depletion at the end of this period equals the pre-specified depletion. This implies that the historical catches (which are, of course, integer numbers) differ among simulations.

Evaluating *Strike Limit Algorithms*
Three candidate *SLA*s are evaluated for the 14 scenarios:

(1) A constant catch strategy - this strategy sets the strike limit each year to the level of (integer) catch which achieves a median final depletion for trial 1 as close as possible to that for the 'Maximum-likelihood-like' *SLA* below.
(2) The Potential Biological Removals (PBR) strategy (see Appendix D for a brief overview).
(3) The 'Maximum-likelihood-like' *SLA* - a variant of an *SLA* considered for the Bering-Chukchi-Beaufort Seas stock of bowhead whales (see Appendix E for a brief overview).

Each simulation trial involves 250 simulations of a 100-year projection period in which the strike limit is set every fifth year. The data available to the *SLA*s are (unbiased) estimates of 1+ abundance (and their CVs) generated using the protocol applied for Bering-Chukchi-Beaufort Seas stock of bowhead whales (see Section B.1 of Appendix 3 of IWC (2002)). These abundance estimates are generated every fifth year with a CV (if the population was 60% of its pre-exploitation equilibrium level) of 0.25, starting the year before the *SLA* is first applied (i.e. it is assumed that no historical estimates of abundance are available). Additional variance in abundance estimates is ignored for the purposes of these trials. The level of need is taken to be infinite for these trials and the historical annual catches supplied to the *SLA* are taken to be the average (over simulations) catch each year (truncated to the nearest integer).

RESULTS AND DISCUSSION
Basic model features
Simulations with no environmental variability or catastrophic events
Fig. 1 plots expected (i.e. average over years and simulations) catch versus exploitation rate and expected catch versus expected number of recruited (equals 1+) animals for trials 1b, 2b and 3b (pre-exploitation size for the 1+ component of the population, K^{1+}, of 3,000 and $MSYR_{1+}$ rates of 2.5, 1 and 4% respectively). These figures were constructed by projecting an unexploited population forward for 800 years under a variety of levels of intended constant exploitation rate. The results for the last 400 years of the 800-year period for 500 such projections were then averaged to obtain the results for each exploitation rate plotted in Fig. 1. The yield curves in Fig. 1 are sufficiently smooth that it can be concluded that the number of replicates conducted (500) was (more than) adequate to determine the value of *A* reasonably accurately. $MSYL_{1+}$ does not occur at $0.6K^{1+}$ for any of these trials (53-56% for the results reported in Fig. 1). This is, however, to be expected given that the recruitment and maturity ogives differ substantially (Punt, 1996). The ratio of the exploitation rate at which the population is rendered extinct within 800 years to $MSYR_{1+}$ decreases as $MSYR_{1+}$ is increased. This ratio is 4 for $MSYR_{1+} = 1\%$ but less than 2 for $MSYR_{1+} = 4\%$.
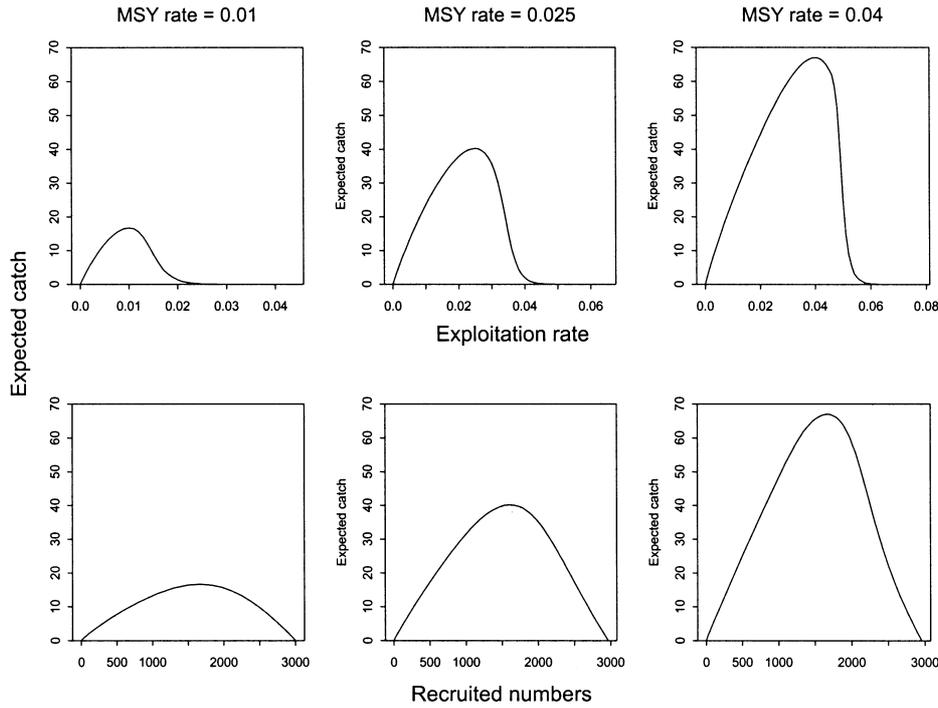
Fig. 1. Expected catch versus exploitation rate and expected number of recruited animals for trials in which $K^{1+} = 3,000$ and $MSYR_{1+} = 1$, 2.5, and 4%.
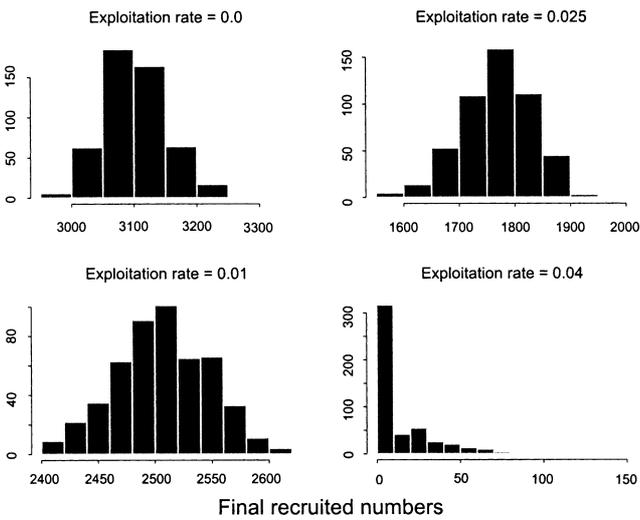

Fig. 2. Distribution of the number of recruited animals after 800 years of the application of a (intended) constant exploitation rate. $MSYR_{1+} = 2.5\%$ and $K^{1+} = 3,000$ for this figure.

Fig. 2 plots the distribution of population size (500 points) after 800 years for trial 1b ($MSYR_{1+} = 2.5\%$; $K^{1+} = 3,000$; population initially at K^{1+}) for fixed exploitation rates of 0, 1, 2.5 and 4% while Fig. 3 shows the time-trajectories of 1+ population size for the first 10 (of 500) simulations for each of these exploitation rates for this trial.

As expected (e.g. May *et al.*, 1978; Sissenwine *et al.*, 1988), Fig. 2 indicates that the variability in the distributions of the number of recruited animals after 800 years increases with increasing exploitation rate (CVs for the final population size of 1.6% for exploitation rates of 0 and 1%, 3.4% for an exploitation rate of 2.5%, and >100% for an exploitation rate of 4%).

The time-trajectories of 1+ population size reach approximate equilibrium after about 100 years, except when the exploitation rate is 4% when the resource is predicted to

be rendered extinct eventually (Fig. 3). It should be noted that although there is variability among simulations (Fig. 2), this variability is not particularly large in comparison to the impact of the harvest (Fig. 3).

Simulations with environmental variability and catastrophic events

Fig. 4 shows 800-year time-trajectories of 1+ population size for the base-case population model (trial 1b) and variants thereof in which allowance is made for catastrophic events, (correlated) environmental variation in survival, and (correlated) environmental variation in births (trials 1b, 5b, 6b and 7b). The value of K^{1+} equals 3,000, $MSYR_{1+} = 2.5\%$, the exploitation rate is 2.5% and the population is initially at K^{1+} for all of the results reported in Fig. 4.

The factor that has the largest impact (i.e. the largest difference from the base-case) is environmental variation in the survival rate. In contrast, the one factor included in the current trials for fishery type 2 (environmental variation in births) has relatively the lowest of the impacts in Fig. 4 (except, of course, for the base-case which does not include any sources of environmental variation). It is noteworthy that when there is environmental variation in survival, the resource is almost rendered extinct on occasion even though the harvest rate equals $MSYR$. The fact that environmental variation in survival has the largest impact in Fig. 4 may appear surprising given the arguments of Mangel and Tier (1994) and Gerber and Hilborn (2001) that catastrophic events are often the most important determinants of the probability of population extinction. However, this can be attributed to the specific choices for the values for the parameters that determine environmental variation of survival and the impact (and frequency) of catastrophic events (for which there is little basis in data). The fact that environmental variation in births has the least impact is probably a fairly general result because environmental variation in survival and catastrophes impact several age-classes whereas the impact of environmental variation in births is restricted to calves.
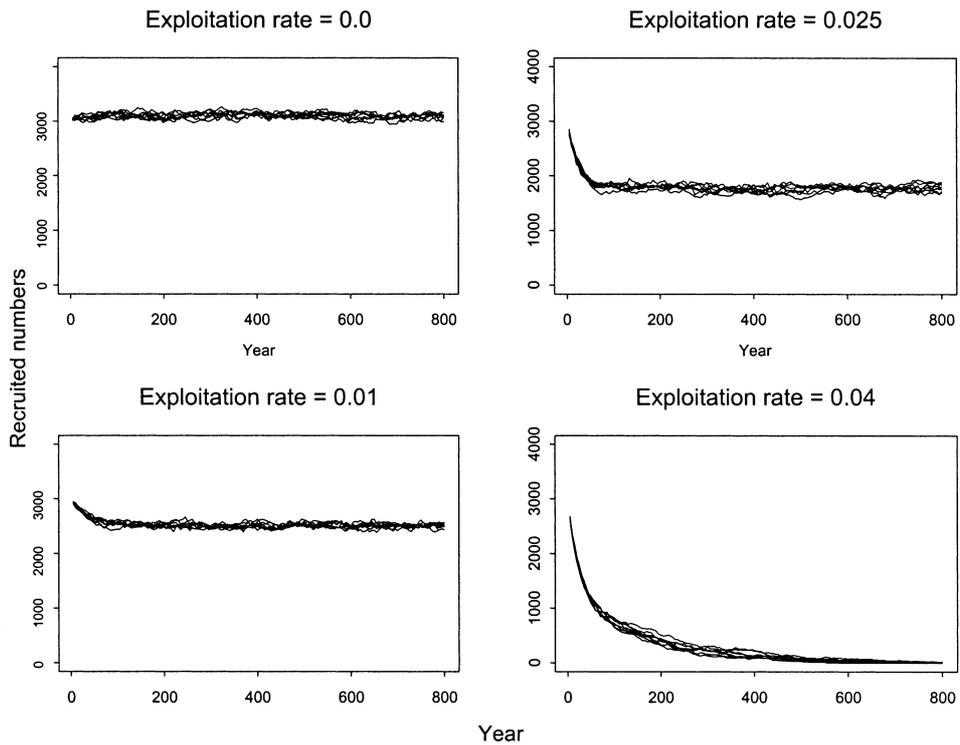
Exploitation rate = 0.0

Exploitation rate = 0.025

Exploitation rate = 0.01

Exploitation rate = 0.04

Fig. 3. Time-trajectories of 1+ population size for ten simulations for four exploitation rates when $K^{1+} = 3,000$ and $MSYR_{1+} = 2.5$.

Base-case

Catastrophic events

Stochastic variation in survival
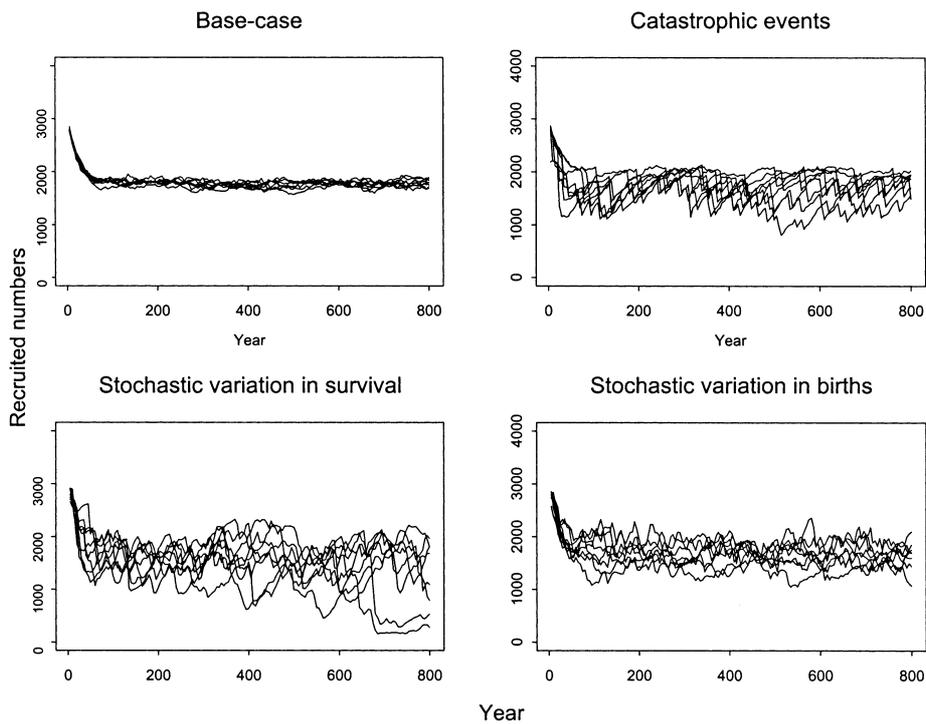
Stochastic variation in births

Fig. 4. Ten time-trajectories of 1+ population size under an exploitation rate of 2.5% based on different amounts (and types) of environmental variability when $K^{1+} = 3,000$ and $MSYR_{1+} = 2.5$.

Fig. 5 contrasts the implications of different levels for the coefficient of variation (CV) of the probability of survival and the extent of inter-annual correlation in annual survival. The 1+ population size is 300 animals (10% of K^{1+}) at the start of the 100-year projection period and no catches are taken for the entire 100 years. The range for the CV of survival is bounded above by 0.1 because, given the specifications related to survival (Table 1), there is no solution to Equation (A.3) for CVs larger than 0.1. The variability in the trajectories increases with inter-annual correlation in survival and with the CV for the probability of survival. In 250 simulations over 100 years, no extinctions occurred for CVs < 0.075 but several extinctions occurred for CVs of 0.075 and larger. The population size after 100 years substantially exceeds K^{1+} in some instances and the frequency of this increases with increasing CV. This occurs because, when the CV of survival is high and given that survival is bounded above by 1, the median probability of survival exceeds the mean probability of survival by an amount that increases as the CV increases. Therefore,
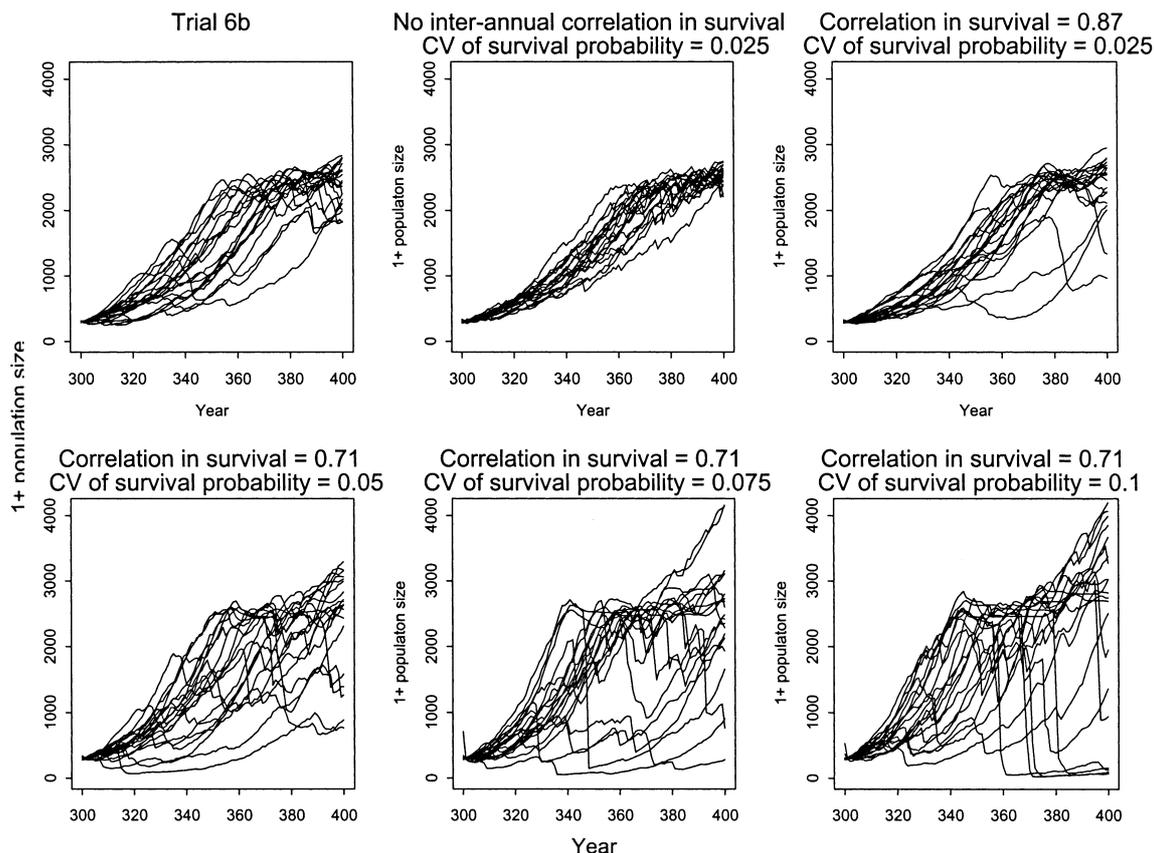
Fig. 5. Twenty 100-year time-trajectories of 1+ population size under zero harvest for various specifications related to the extent of environmental variation in survival for the scenario in which the population is depleted to 300 animals (10% of K^{1+}) at the start of year 300.

although the expected population size in the absence of exploitation is K^{1+}, the higher the variability in the probability of survival, the more substantially the population will exceed K^{1+} on occasion.

Comparison with a lumped model

A deterministic lumped model cannot mimic the types of behaviour evident in Figs 4 and 5 for the model variants in which there is environmental variability in survival or in births. In contrast, the level of variability due solely to 'demographic stochasticity' (see Fig. 3 and top left panel of Fig. 4) is sufficiently small that the results of a deterministic model would not be qualitatively different.

It would be difficult to implement some of the features of the individual-based model within a standard lumped model (for example, the assumption that if a calf dies, its mother is more likely to calve immediately, and, particularly, that the catch is taken multinomially from the population). An additional feature that would be difficult to implement using a standard lumped model would be the regulation that mothers with calves cannot be harvested. In addition, the use of an individual-based model as the operating model ensures that the number of animals (for each age and sex) is always an integer; the lumped model developed by IWC (2002) does not ensure that this is the case.

The individual-based model framework is therefore clearly more flexible in terms of the assumptions that it can represent. However, this flexibility is not without (computational) cost. In particular, the software required to implement the individual-based model is substantially more complex than that to implement the corresponding lumped model, has markedly larger storage requirements (it is

necessary to store several pieces of information for each animal) and runs considerably more slowly. The impact of the last two disadvantages increases with the number of animals. For example, the time required to evaluate *SLA*s using operating models 4a and 4b is at least an order of magnitude longer than is the case for the other operating models. However, for type 3 fisheries, the population size is relatively small by definition, so that these computational constraints are not as prohibitive as might be the case if the individual-based model formed the basis for an operating model for, say, the eastern North Pacific stock of gray whales for which the current population size is over 20,000.

While it is clear that deterministic lumped models are inappropriate as the basis for operating models for type 3 fisheries, it may well be case that models that pool individuals of a given age and maturity state (could give birth this year, gave birth last year, etc.) and allow for demographic and environmental stochasticity in births and deaths could mimic the behaviour of an individual-based model adequately. Evaluation of this issue is, however, beyond the scope of the current paper but should form a focus for future work.

Evaluation of *SLA*s

Table 3 contrasts the performances of the three *SLA*s described above and the strategy of setting zero strike limits, in terms of a subset of the performance statistics used to evaluate *SLA*s for other aboriginal whaling operations (e.g. IWC, 2002). The performance statistics are the 5th, median and 95th points of the distributions for the following quantities:

Table 3

Performance statistics for four alternative *SLA*s for the 14 simulation trials.

(a) Zero catches for 100 years.

Description	Trial no.	Final depletion (1+)			Final depletion (mat)			Lowest depletion (1+)			Relative recovery (1+)			Relative recovery (mat)		
		5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%
Base-case	1a	0.752	0.799	0.825	0.899	1.001	1.058	0.045	0.049	0.054	14.7	16.0	17.5	16.3	18.4	21.2
	1b	0.816	0.845	0.871	0.993	1.035	1.077	0.095	0.100	0.104	8.0	8.5	8.9	8.5	9.5	11.1
Low MSYR$_{1+}$	2a	0.149	0.200	0.256	0.160	0.222	0.290	0.045	0.050	0.054	3.0	4.0	5.0	3.1	4.0	5.3
	2b	0.302	0.392	0.505	0.322	0.435	0.592	0.095	0.099	0.104	3.0	3.9	5.1	3.0	4.0	5.1
High MSYR$_{1+}$	3a	0.788	0.819	0.846	0.987	1.010	1.035	0.046	0.050	0.054	15.1	16.5	17.9	17.3	19.9	23.2
	3b	0.865	0.901	0.943	0.941	0.974	1.009	0.095	0.100	0.104	8.5	9.0	9.6	8.5	9.5	10.9
Large population size	4a	0.790	0.804	0.813	0.978	1.011	1.033	0.046	0.051	0.054	14.9	15.8	17.1	17.2	18.2	19.7
	4b	0.836	0.846	0.856	1.021	1.033	1.045	0.096	0.101	0.104	8.1	8.4	8.8	9.0	9.5	10.1
With catastrophic events	5a	0.572	0.809	0.843	0.650	1.015	1.101	0.043	0.049	0.054	11.8	15.8	18.0	12.9	18.3	22.0
	5b	0.671	0.842	0.908	0.775	1.044	1.102	0.087	0.099	0.104	6.7	8.4	9.2	7.0	9.4	10.8
With env. variation in survival	6a	0.355	0.789	0.878	0.401	0.978	1.158	0.044	0.049	0.054	7.4	15.5	18.3	7.7	17.7	23.1
	6b	0.585	0.840	0.942	0.624	0.997	1.160	0.086	0.099	0.104	6.0	8.5	9.5	6.0	9.3	11.4
With env. variation in births	7a	0.664	0.777	0.889	0.787	0.975	1.103	0.045	0.050	0.054	13.1	15.4	18.0	14.0	17.7	20.9
	7b	0.738	0.828	0.940	0.897	1.010	1.124	0.095	0.099	0.104	7.3	8.3	9.5	7.7	9.3	10.9

(b) The 'Maximum likelihood-like' *SLA*.

Description	Trial no.	Final depletion (1+)			Final depletion (mat)			Lowest depletion (1+)			Relative recovery (1+)			Relative recovery (mat)			Average annual catch			AAV		
		5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%
Base-case	1a	0.074	0.558	0.808	0.077	0.641	1.028	0.037	0.049	0.054	1.6	11.2	17.1	1.8	11.8	19.6	0.2	10.0	21.4	0.039	0.084	0.720
	1b	0.385	0.690	0.868	0.445	0.805	1.060	0.095	0.100	0.104	4.0	6.8	8.8	4.1	7.5	10.4	0.2	11.9	19.1	0.033	0.053	0.800
Low MSYR$_{1+}$	2a	0.011	0.177	0.235	0.013	0.197	0.268	0.004	0.049	0.054	0.2	3.6	4.7	0.2	3.6	4.8	0.2	0.3	5.2	0.103	0.322	0.800
	2b	0.168	0.348	0.477	0.189	0.382	0.547	0.092	0.098	0.104	1.7	3.5	4.8	1.7	3.5	4.7	0.1	0.2	4.2	0.077	0.350	0.800
High MSYR$_{1+}$	3a	0.017	0.716	0.828	0.018	0.811	1.028	0.013	0.049	0.054	0.3	14.3	17.5	0.4	15.8	22.4	0.3	40.3	57.4	0.029	0.041	0.771
	3b	0.622	0.716	0.903	0.662	0.804	0.997	0.095	0.099	0.104	6.2	7.2	9.3	6.4	7.8	10.0	0.3	34.5	39.0	0.025	0.034	0.633
Large population size	4a	0.071	0.226	0.780	0.076	0.249	0.966	0.032	0.049	0.054	1.4	4.4	15.8	1.4	4.5	17.9	11.5	139.4	203.3	0.033	0.047	0.211
	4b	0.364	0.625	0.833	0.402	0.719	1.035	0.096	0.101	0.104	3.6	6.2	8.0	3.7	6.6	9.2	4.2	139.3	172.2	0.034	0.044	0.362
With catastrophic events	5a	0.039	0.595	0.843	0.042	0.687	1.093	0.009	0.051	0.089	1.4	10.4	22.6	1.5	11.7	25.2	0.2	0.5	30.1	0.037	0.240	0.760
	5b	0.272	0.701	0.876	0.292	0.794	1.091	0.042	0.096	0.186	2.1	6.3	15.5	2.1	6.7	18.2	0.2	10.4	25.4	0.033	0.060	0.743
With env. variation in survival	6a	0.036	0.547	0.859	0.039	0.619	1.132	0.005	0.045	0.155	0.9	9.1	30.6	1.0	9.8	33.8	0.2	0.6	45.2	0.037	0.171	0.829
	6b	0.185	0.671	0.882	0.205	0.772	1.126	0.021	0.101	0.282	1.7	5.6	16.8	1.7	6.1	21.6	0.2	7.6	28.6	0.031	0.077	0.850
With env. variation in births	7a	0.018	0.612	0.843	0.019	0.715	1.093	0.006	0.048	0.082	0.6	11.9	20.2	0.6	13.0	22.4	0.2	0.7	27.4	0.038	0.216	0.771
	7b	0.332	0.664	0.901	0.390	0.783	1.075	0.057	0.102	0.160	3.2	6.1	12.2	3.1	6.7	14.0	0.2	11.2	20.8	0.035	0.055	0.880

(c) Constant catch strategy

Description	Trial no.	Final depletion (1+)			Final depletion (mat)			Lowest depletion (1+)			Relative recovery (1+)			Relative recovery (mat)			Average annual catch		
		5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%
Base-case	1a	0.477	0.652	0.749	0.531	0.756	0.918	0.045	0.049	0.054	10.0	13.1	14.8	10.2	14.0	16.9	5.0	5.0	5.0
	1b	0.725	0.790	0.820	0.861	1.003	1.052	0.094	0.099	0.104	7.3	7.9	8.4	8.1	9.1	10.3	5.0	5.0	5.0
Low $MSYR_{1+}$	2a	0.003	0.042	0.090	0.004	0.054	0.113	0.003	0.039	0.052	0.1	0.8	1.8	0.1	1.0	2.0	4.9	5.0	5.0
	2b	0.003	0.078	0.182	0.007	0.101	0.227	0.003	0.073	0.100	0.0	0.8	1.8	0.1	0.9	1.9	4.8	5.0	5.0
High $MSYR_{1+}$	3a	0.761	0.786	0.814	0.996	1.017	1.037	0.045	0.050	0.054	14.5	15.8	17.2	17.5	20.0	23.3	5.0	5.0	5.0
	3b	0.792	0.836	0.876	0.949	0.987	1.025	0.095	0.099	0.104	7.9	8.4	8.8	8.5	9.7	11.0	5.0	5.0	5.0
Large population size	4a	0.780	0.799	0.810	0.957	0.999	1.022	0.046	0.051	0.054	14.8	15.7	16.9	17.0	18.1	19.4	5.0	5.0	5.0
	4b	0.832	0.841	0.851	1.024	1.035	1.048	0.096	0.101	0.104	8.1	8.4	8.8	9.0	9.5	10.1	5.0	5.0	5.0
With catastrophic events	5a	0.031	0.623	0.827	0.044	0.734	1.082	0.017	0.051	0.089	1.6	10.6	16.7	1.8	11.9	18.6	5.0	5.0	5.0
	5b	0.000	0.754	0.882	0.000	0.913	1.078	0.000	0.096	0.186	0.0	6.6	10.0	0.0	7.4	11.9	4.1	5.0	5.0
With env. variation in survival	6a	0.000	0.596	0.874	0.000	0.685	1.139	0.000	0.045	0.151	0.0	8.1	19.5	0.0	9.1	22.9	1.4	5.0	5.0
	6b	0.000	0.744	0.946	0.000	0.862	1.134	0.000	0.099	0.282	0.0	5.1	11.6	0.0	5.5	14.4	1.2	5.0	5.0
With env. variation in births	7a	0.106	0.630	0.851	0.121	0.747	1.049	0.027	0.049	0.082	3.5	10.9	16.1	3.7	12.3	17.3	5.0	5.0	5.0
	7b	0.316	0.762	0.888	0.362	0.950	1.091	0.056	0.102	0.157	4.2	7.0	9.5	4.7	8.0	11.0	5.0	5.0	5.0

(d) PBR approach

Description	Trial no.	Final depletion (1+)			Final depletion (mat)			Lowest depletion (1+)			Relative recovery (1+)			Relative recovery (mat)			Average annual catch			AAV		
		5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%	5%	Median	95%
Base-case	1a	0.499	0.603	0.697	0.561	0.693	0.832	0.045	0.049	0.054	10.2	12.2	13.8	10.5	12.8	15.4	7.7	9.9	12.8	0.070	0.103	0.140
	1b	0.712	0.759	0.784	0.821	0.924	0.984	0.095	0.100	0.104	7.1	7.6	8.0	7.5	8.5	9.6	7.4	8.9	10.5	0.063	0.088	0.117
Low $MSYR_{1+}$	2a	0.084	0.117	0.156	0.090	0.132	0.182	0.044	0.049	0.054	1.7	2.3	3.2	1.6	2.4	3.4	1.9	2.8	3.6	0.133	0.185	0.265
	2b	0.170	0.231	0.309	0.191	0.266	0.360	0.093	0.098	0.104	1.7	2.3	3.1	1.8	2.4	3.2	2.2	3.0	4.0	0.097	0.138	0.188
High $MSYR_{1+}$	3a	0.759	0.783	0.808	0.951	0.979	1.006	0.045	0.050	0.054	14.4	15.8	17.2	16.8	19.3	22.5	16.4	19.2	22.8	0.056	0.080	0.102
	3b	0.785	0.816	0.852	0.927	0.963	1.004	0.095	0.099	0.104	7.7	8.2	8.7	8.4	9.4	10.7	11.2	12.4	14.0	0.059	0.078	0.102
Large population size	4a	0.499	0.577	0.630	0.559	0.656	0.728	0.046	0.051	0.054	10.0	11.3	12.4	10.4	12.0	13.2	86.2	101.1	115.5	0.072	0.095	0.133
	4b	0.719	0.749	0.770	0.852	0.899	0.937	0.096	0.101	0.104	7.1	7.5	7.9	7.7	8.3	8.8	78.0	87.6	98.0	0.062	0.082	0.108
With catastrophic events	5a	0.356	0.621	0.760	0.385	0.715	0.940	0.043	0.049	0.054	7.2	12.4	15.4	7.2	13.0	17.5	6.0	10.5	14.7	0.068	0.097	0.152
	5b	0.517	0.751	0.817	0.572	0.904	1.021	0.085	0.099	0.104	5.1	7.5	8.3	5.5	8.2	9.8	5.7	8.6	11.0	0.064	0.089	0.124
With env. variation in survival	6a	0.224	0.618	0.819	0.253	0.717	1.074	0.042	0.049	0.054	4.7	12.2	16.9	4.7	13.0	20.2	4.5	10.1	17.5	0.067	0.106	0.170
	6b	0.393	0.750	0.846	0.450	0.889	1.103	0.081	0.098	0.104	4.0	7.5	8.6	4.2	8.2	10.6	4.3	9.2	11.9	0.060	0.088	0.134
With env. variation in births	7a	0.429	0.591	0.730	0.477	0.680	0.855	0.045	0.050	0.054	8.7	11.8	14.6	8.9	12.4	15.9	7.2	10.0	13.3	0.069	0.097	0.143
	7b	0.609	0.724	0.846	0.707	0.890	1.020	0.094	0.099	0.104	6.2	7.2	8.6	6.3	8.1	9.7	6.5	8.5	10.4	0.064	0.087	0.118

(1) the final depletion - the ratio of the population size (1+ and mature) at the end of the projection period to the average pre-exploitation level.
(2) the lowest depletion – the ratio of the lowest 1+ population size during the projection period to the average pre-exploitation level.
(3) the relative recovery – the ratio of the population size (1+ and mature) at the end of the projection period to that at the start of this period.
(4) the average annual catch.
(5) the average absolute variation (AAV) in annual catch.

A variety of other performance statistics are used by the IWC Scientific Committee to evaluate *SLA*s. However, these statistics are not currently easily amenable to simulation trials based on individual-based models.

The values for the performance statistics all indicate that, in the absence of future catches, the population size after 100 years will exceed that at the start of the projection period (Table 3a). The extent of increase (as measured by the 'relative recovery' statistics) differs depending on the *MSYR* and the initial state of the resource (5 or 10% of the average pre-exploitation level). The final depletions are, as expected, lower if the strike limits are set using the 'Maximum-likelihood-like' *SLA* (Table 3b). However, some recovery is guaranteed in many of the cases. The exceptions to this (all of which have an initial depletion of 0.05) are trials 2a (initial depletion = 0.05; $MSYR_{1+} = 1\%$), 3a (initial depletion = 0.05; $MSYR_{1+} = 4\%$), 6a (initial depletion = 0.05; environmental variation in survival), and 7a (initial depletion = 0.05; environmental variation in births) for which the lower 5th percentiles for the relative recovery statistic are less than unity. The reasons for the poor performance for trials 2a, 6a and 7a are readily apparent: these are cases in which productivity is poor or survival is subject to environmental stochasticity. The poor performance for trial 3a is surprising; this is a case in which productivity is high, and substantial resource recovery occurs in the bulk of the simulations. It would seem to be the consequence of a few large catch limits in the first years of the 100-year period leading to a high exploitation rate at low population size.

The value of applying a feedback harvest strategy can be evaluated by comparing the results in Tables 3b and 3c. The constant level of catch for Table 3c was chosen so that the median final depletion for trial 1 was as close as is possible (given that the strike limit has to be an integer) to that for the 'Maximum-likelihood-like' *SLA*. This level of constant catch is 5 and, in fact, the median final depletion for trial 1 substantially exceeds that for 'Maximum-likelihood-like' *SLA*. However, the performance of the constant catch *SLA* is very poor (for example, extinction occurs in more than 5% of simulations for trials 5b, 6a and 6b and more than 50% of the final depletions for trials 2a and 2b are below the initial depletions), which indicates that there is considerable value in applying a feedback harvest strategy for type 3 fisheries.

The PBR approach (Table 3d) is inherently more conservative than the 'Maximum-likelihood-like' *SLA* (note the generally lower average catches/strikes). Consequently, this *SLA* does not lead to values for the lower 5th percentile of the relative recovery statistic less than unity for any of the trials. More importantly, the PBR approach does not drop the resource appreciably below the level that would be achieved in the absence of any catches for trials 5a-7b (contrast the lower 5thpercentiles of the lowest depletion distributions in Tables 3a and 3d).

Worked needed to tailor the framework to specific type 3 fisheries
The information required to develop case-specific trials for actual type 3 fisheries includes the following:

(1) historical catches.
(2) biological parameters (survival rates, fecundity rates, minimum calving interval).
(3) extent of environmental variability in survival and birth rate.
(4) probability and expected severity of catastrophic events.
(5) nature and frequency of future data collection programmes.
(6) relative (age- and sex-specific) probability of being harvested.
(7) scenarios regarding *MSYR* and *MSYL*.
(8) the bounds on the levels of need that an *SLA* will have to be able to cope with.
(9) performance statistics.

The information for type 3 fisheries tends to be even worse than is the case for type 1 and type 2 fisheries. This may mean that instead of being able to specify values for some of the parameters of the operating model exactly, ranges for the values for these parameters will have to be considered. One implication of this is that the range of scenarios that may need to be considered may be very wide (to ensure that the actual situation is covered by the set of trials). The extent of environmental variation in birth and survival rates and the probability and severity of catastrophic events cannot be estimated for any type 3 fisheries. However, it is possible that inferences from other whale species (e.g. Angliss *et al.*, 1995; Perryman *et al.*, 1997) could be used to develop ranges for, for example, the extent of environmental variation in births. Developing scenarios regarding catastrophic events using data for cetaceans will be even more problematic than developing such scenarios for the extent of environmental variation in birth and survival rates. Gerber and Hilborn (2001) provide data related to catastrophic events for a range of otariid species and it may be possible to utilise information of this type when developing scenarios.

The primary data available for type 1 and 2 fisheries on which strike limits might be based has been assumed to be unbiased estimates of total (1+) abundance. Unfortunately, the absolute abundance data for some type 3 fisheries are likely to comprise only very infrequent (and possibly highly biased) estimates of population size. For such cases, it may be necessary to use the information on relative abundance and from the age-structure of the catch to achieve reasonable *SLA* performance.

The scenarios for *MSYR* could be expressed in terms of the expected rate of recovery at very low stock size. Values for this rate could plausibly be inferred from data for other whale species (e.g. Best (1993)).

The specification of a need envelope (i.e. bounds on possible levels of need) would have to be achieved through discussions at the Commission, with technical advice from the Scientific Committee. The selection of appropriate tunings of candidate *SLA*s would need to be coordinated between the IWC Scientific Committee and the Commission.

General discussion
The framework in this paper could be extended substantially depending on circumstances. There are several areas where additional model development work may be required for

some type 3 fisheries, for example: Allee effects; time-dependence in catastrophic events; catastrophic impacts on the birth rate; changes over time in the environment and spatial structure. The first of these has been ignored in the analyses of this paper because, although there must be a 'level below which recovery is impossible', there are too few data to define this level generically. In fact, there may be some good reasons not to define this level but (as was the case during the development of the RMP) to instead interpret performance on the basis of the lowest depletion statistic. The individual-based model can be extended to include spatial structure (as has been done for lumped models for the *Implementation Simulation Trials* developed for the North Atlantic, Southern Hemisphere and North Pacific minke whales). However, as was the case for the RMP, it would seem advisable to first examine the generic properties of *SLA*s for type 3 fisheries before submitting them to evaluation against detailed spatially structured case-specific trials.

The process of conditioning case-specific trials may be substantially more complicated if operating models for type 3 fisheries are to be founded on individual-based models. For example, the approach used to condition the trials for the Bering-Chukchi-Beaufort Seas stock of bowhead whales and of those for the eastern North Pacific stock of gray whales involves placing a prior on the current population size. Unfortunately, it is not easy to 'solve' an individual-based model for K such that the current abundance (or depletion) equals a particular value. In this paper, the constraint that the depletion of the 1+ component of the population after the period of historical catches equals 0.05 or 0.1 was achieved subject to a tolerance of 0.005. This aspect of the conditioning problem could be overcome by placing a prior on K rather than on current depletion or current abundance, although this approach to conducting Bayesian assessments has been criticised in the past in the Scientific Committee (e.g. Butterworth and Punt, 1995).

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Appendix A

GENERATING THE AGE- AND SEX-SPECIFIC SURVIVAL RATES

The probability that an animal of sex s and age a survives the impact of natural mortality, $S^s_{y,a}$, when subject to temporally correlated environmental variation and to occasional 'catastrophic' reductions in survival, is given by:

$$S^s_{y,a} = \frac{e^{q^s_{y,a}}}{e^{q^s_{y,a}}+1} U_y \qquad q^s_{y,a} = \mu^s_a + \rho(q^s_{y-1,a} - \mu^s_a) + \varepsilon_y \sigma^s_a \quad \text{(A.1)}$$

where:

$S^s_{y,a}$ is the survival probability for animals of sex s and age a during year y;

$q^s_{y,a}$ is the (logit-transformed) survival probability for animals of sex s and age a during year y in the absence of catastrophic events;

μ^s_a is the mean of the distribution for $q^s_{y,a}$;

ρ is the temporal correlation in the environmental anomalies in survival rate;

σ^s_a is the standard deviation of the survival anomalies;

ε_y is a random variate generated from $N(0;1^2)$;

U_y is a factor to account for (temporally independent) 'catastrophic' reductions in survival rate:

$$U_y = \begin{cases} U_{low} & \text{if } \tilde{\varepsilon}_y < p_{low} \\ (1 - U_{low}\, p_{low})/(1 - p_{low}) & \text{otherwise} \end{cases} \quad \text{(A.2)}$$

U_{low} is the fraction by which survival is reduced if a 'catastrophic' event takes place;

p_{low} is the probability of a 'catastrophic' event taking place; and

ε_y is a random variate generated from U[0,1].

The values for the μ^s_as and σ^s_as are chosen so that the expected survival rate and the coefficient of variation of survival rate equal pre-specified values — $E(S^s_a)$ and $CV(S^s_a)$ respectively. This involves choosing values for μ^s_a and σ^s_a to satisfy the following system of equations:

$$E(S^s_a) = \int_{-\infty}^{\infty} \frac{e^q}{1+e^q} \frac{\sqrt{1-\rho^2}}{\sqrt{2\pi}\,\sigma^s_a} e^{-\frac{1-\rho^2}{2(\sigma^s_a)^2}(q-\mu^s_a)^2} dq$$

$$CV(S^s_a)$$
$$= \frac{1}{E(S^s_a)} \sqrt{\int_{-\infty}^{\infty} \left(\frac{e^q}{1+e^q}\right)^2 \frac{\sqrt{1-\rho^2}}{\sqrt{2\pi}\,\sigma^s_a} e^{-\frac{1-\rho^2}{2(\sigma^s_a)^2}(q-\mu^s_a)^2} dq - E(S^s_a)^2}$$
$$\text{(A.3)}$$

The formulation for U_y is selected so that the expected value of U_y equals 1.

Appendix B

DETERMINING THE BIRTH RATE PROBABILITY

The expected probability during year y that a mature female that has not given birth for at least x years gives birth, $E(f_y)$ is given by:

$$E(f_y) = f_0(1 + A(1 - (P^{1+}_y / P^{1+}_{-\infty})^z)) \quad \text{(B.1)}$$

where:

f_0 is the birth rate at the average pre-exploitation level;

A is the 'resilience' parameter;

z is the 'degree of compensation' parameter;

P^{1+}_y is the number of 1+ animals at the start of year y; and

$P^{1+}_{-\infty}$ is the (average) number of 1+ animals in an unexploited state.

The value of f_0 is given by:

$$f_0 = \frac{B_{-\infty} / N^m_{-\infty}}{1 - \frac{1}{N^m_{-\infty}} \sum_{y=2}^{x} B_{-\infty}\, e^{-M(y-2)}} \quad \text{(B.2)}$$

where:

$B_{-\infty}$ is the (average) number of births (calves) in an unexploited state;

x is the minimum calving interval;

M is the instantaneous rate of natural mortality on mature animals; and

$N^m_{-\infty}$ is the (average) number of mature animals in an unexploited state.

The realised value for the probability during year y of a mature female that has not given birth for at least x years giving birth is generated using the equation:

$$f_y = \frac{e^{\hat{q}_y}}{e^{\hat{q}_y}+1} \qquad \hat{q}_y = \mu_y + \tilde{\rho}(\hat{q}_{y-1} - \mu_{y-1}) + \eta_y \sigma_F \quad \text{(B.3)}$$

where:

μ_y is selected (G. Givens, pers. comm.) so that:

$$E(f_y) = \int_{-\infty}^{\infty} \frac{e^q}{1+e^q} \frac{\sqrt{1-\tilde{\rho}^2}}{\sqrt{2\pi}\,\sigma_F} e^{-\frac{1-\tilde{\rho}^2}{2(\sigma_F)^2}(q-\mu_y)^2} dq \quad \text{(B.4)}$$

σ_F is a measure of the environmental variation in births;

$\tilde{\rho}$ is the temporal correlation in birth rate; and

η_y is a random variate generated from $N(0;1^2)$.

Appendix C

SOLVING FOR A GIVEN $MSYR$

$MSYR$ can be defined by the equation:

$$\left.\frac{dC}{dF}\right|_{F=MSYR} = 0 \qquad \text{(C.1)}$$

where $C(F)$ is the catch (expected catch for a stochastic model) as a function of the intended exploitation rate F.

A numerical approach for computing A given $MSYR$ is therefore to search (using, for example, a bisection method) for the value of A such that

$$C(MSYR + \Delta F) - C(MSYR - \Delta F) = 0.$$

This is a straightforward calculation for deterministic models for which the function $C(F)$ is well defined (e.g. Punt (1996; 1999)). Unfortunately, this is not the case for stochastic models. For such models, it is necessary to define $C(F)$ as the average of the catch when the exploitation rate is fixed at F. In this paper, $C(F)$ for a single replicate is obtained by projecting from the average pre-exploitation level for 800 years fixing the exploitation rate to F. The average catch over the last 400 years of the 800-year projection period is then taken to be $C(F)$ for that replicate. The choice of 400 years for the 'burn in' period was selected based on projections for the base-case assumptions under a range of values for the intended exploitation rate, F.

Appendix D

THE PBR APPROACH

The strike limit for year y, Q_y, is defined according to the equation:

$$Q_y = F_R\, 0.5\, R_{\max}\, N_{\min}(y) \qquad \text{(D.1)}$$

where:

F_R is the 'recovery factor' – assumed to be 0.5 for the analyses of this paper;

$R_{\max}$ is the maximum theoretical net productivity rate (assumed to be 0.04 – the default value for cetaceans – Wade, 1998); and

$N_{\min}(y)$ is a 'minimum population size' (the lower 20th% confidence limit) based on the most recent estimate of abundance.

Appendix E

THE 'MAXIMUM-LIKELIHOOD' SLA

The population dynamics model and likelihood function that form the basis for this SLA are:

$$N_{y+1} = N_y + 1.4184\, MSYR\, N_y(1 - (N_y/K)^2) - C_y \,^6 \qquad \text{(E.1)}$$

$$L = \prod_{y<y^\bullet} \exp\left(-\frac{1}{2}\frac{\ell n(N_y/N_y^{obs})^2}{(CV_y^{obs})^2 + \{CV_{add}(y-y^\bullet)/10\}^2}\right) \qquad \text{(E.2)}$$

where:

N_y is the number of (1+) animals (of both sexes) at the start of year y;

K is the pre-exploitation number of 1+ animals;

$MSYR$ is the MSY rate parameter;

C_y is the catch (both sexes) during year y;

$y^\bullet$ is the year for which a strike limit is required;

N_y^{obs} is the estimate of 1+ abundance for year y;

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[6] This population dynamics model is a modification of a discrete logistic model with the maximum rate of increase parameterised in terms of $MSYR$.

CV_y^{obs} is the observed coefficient of variation for N_y^{obs}; and

CV_{add} is the 'additional' coefficient of variation – defined as the CV added to that for surveys conducted ten years before the year for which a strike limit is required.

The calculation of a strike limit for year $y^\bullet$ involves first calculating a 'raw' strike limit for year $y^\bullet$, Q_y^R, and modifying this to conform with constraints on inter-5-year-block variability in strike limits. The 'raw' strike limit for year $y^\bullet$ is computed using the following algorithm.

(1) Find, for 50 values for $MSYR$ spaced equally between 0 and 0.05, the values for K that maximise Equation (E.2).

(2) Assign weights to each of the 50 combinations of $MSYR$ and K obtained at step (1) using the formula $\exp(-likelihood)\, P(MSYR)$ where $likelihood$ is the negative log-likelihood and the prior is of the form:

$P(MSYR) =$

$$\begin{cases} P_{\max} & \text{if } MSYR \leq 0.01 \\ P_{\max} + \frac{1-P_{\max}}{0.02}(MSYR - 0.01) & \text{if } 0.01 < MSYR < 0.03 \\ 1 & \text{if } MSYR \geq 0.03 \end{cases}$$

$$\text{(E.3)}$$

where $P_{\max}$ is the maximum of the 'prior' for *MSYR*.

These 50 combinations of *MSYR* and *K* with their corresponding weights (normalised to sum to 1) form the 'posterior' distribution for steps (3) and (4).

(3) Find the strike limit that satisfies the criterion that Q_1^{th} 'posterior' percentile of $P_{y^{\bullet}+20}/K$ equals a pre-specified value *MSYL* (the value for $P_{y^{\bullet}+20}/K$ is computed by projecting the model 20 years into the future assuming that catch is constant from year $y^{\bullet}$ to year $y^{\bullet} +20$).

(4) Find the strike limit that satisfies the criterion that the Q_2^{th} posterior percentile of $P_{y^{\bullet}+20}/P_{y^{\bullet}}$ equals a pre-specified value $Z_{2,crit}$.

The constraints on inter-block variability in strike limits are imposed using the formula:

$P(MSYR) =$

$$\begin{cases} P_{\max} & \text{if } MSYR \leq 0.01 \\ P_{\max} + \frac{1-P_{\max}}{0.02}(MSYR - 0.01) & \text{if } 0.01 < MSYR < 0.03 \\ 1 & \text{if } MSYR \geq 0.03 \end{cases}$$

$$\text{(E.4)}$$

where:

$Q_{y^{\bullet}}^{\bullet}$ is the strike limit for year $y^{\bullet}$ following modification to constrain variability in strike limits; and

β_1, β_2 are the parameters that determine the constraints on inter-block variation in catch limits.

The values for the tuning parameters are taken to be $P_{\max} = 5$, $CV_{add} = 0.025$, $MS\tilde{Y}L = 0.6$, $Z_{2,crit} = 1.1$, $Q_1 = 0.19$, $Q_2 = 0.19$, $\beta_1 = 0.25$, and $\beta_2 = 1.75$.