Investigations of an Aboriginal Whaling Management Procedure using Adaptive Kalman Filtering

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ABSTRACT

The feasibilty of using Kalman Filter methods as the basis for an Aboriginal Whaling Management Procedure is explored in this paper. Adaptive Kalman Filters are used to obtain estimates of the stock size and posterior probability distributions for MSY rate (*MSYR*) and the pre-exploitation stock size *K*. A set of catch control laws is then used on these estimates of stock size, which together with the posterior distributions of the various combinations of *MSYR* and *K*, gives a cumulative distribution function for the strike limit. The eventual strike limit is then determined as a pre-specified percentile of this distribution. The procedure is tested on some *Evaluation Trials* set by the Standing Working Group on Aboriginal Whaling Management Procedures of the International Whaling Commission (IWC) Scientific Committee. The estimation of a bias factor was considered and results are presented.

KEYWORDS: WHALING-ABORIGINAL; MANAGEMENT PROCEDURE; KALMAN FILTER

INTRODUCTION

The Kalman Filter is widely used in the engineering sciences to obtain estimates of the state of a stochastic dynamical system with noisy observations, i.e. a system with both 'process noise' and 'observation noise'. Kalman Filtering techniques have been applied to estimation problems in fisheries management with some success (e.g. Gudmundsson, 1994; Reed and Simons, 1996) to estimate stock sizes and population parameters using effort data and catch-at-age data. It would therefore seem feasible to apply these techniques to estimate the status of whale stocks. The equations used to describe the population dynamics of whales and the relation between the true stock size and observations thereof can be written in a form which lends itself to state estimation via Kalman Filters. Such estimation schemes together with appropriate catch control laws might then form the basis of an Aboriginal Whaling Management Procedure (AWMP). The potential AWMP described here uses the so-called Adaptive Kalman Filter (AKF) to obtain estimates for the present stock size N_t - conditional on fixed values of *MSYR* (MSY-rate), pre-exploitation population size *K* and bias factor *B* - together with posterior probability distribution for N_t and *(MSYR, K, B)* for each point in a three-dimensional grid of discrete values of these three parameters. Associated with each *(MSYR, K, B)* in the grid, there is a corresponding catch control law. A sequence of catch limits with associated probabilities is thereby obtained and hence a cumulative distribution function for the catch limit can be constructed. Fixing a percentile of this distribution then determines the actual catch limit. This method is therefore a combination of Kalman Filtering techniques and Bayesian methodology.

Fishery type 2 as defined in IWC (2000a) represents cases where substantial information exists. An example of such a fishery is the Bering-Chukchi-Beaufort (B-C-B) Seas stock of bowhead whales *(Balaena mysticetus)*. A series of abundance observations exists for this stock and observations will probably continue to be available at regular intervals. An AWMP proposed for a type 2 fishery must undergo a series of simulation trials. These have been designed by the Scientific Committee of the IWC and are given in IWC (2000b). The trials are conditioned on the data for the B-C-B bowhead stock, i.e. on the history of catches, past stock estimates and parameter values. The AWMP proposed here is fairly general and should be applicable to a range of stocks, but the specifications - including model equations and some parameter values - refer to an application to the B-C-B stock.

The next section is a brief review of the ideas underlying the discrete Kalman Filter. For further details, see for example Brown and Hwang (1997) or Gershenfeld (1999). A general framework is then formulated so that these methods can be applied to the case of a whale stock with abundance observations, followed by a fully specified model applicable to the B-C-B stock, for which results for a set of the AWMP simulation trials are presented. Finally, the results of some sensitivity tests are presented and discussed.

A BRIEF REVIEW OF KALMAN FILTERING TECHNIQUES

The Kalman Filter

The Kalman filter is designed to give an estimate of the state of a system:

$$x_{t+1} = F_t x_t + u_t \quad (1)$$

$$z_t = H_t x_t + v_t \quad (2)$$

where

x_t is the state of the system at time *t* (*n*-vector);
z_t is the observation of the system at time *t* (*m*-vector);
u_t is Gaussian white noise, $u_t \sim N(0,Q_t)$ (*n*-vector);
v_t is Gaussian white noise, $v_t \sim N(0,R_t)$ (*m*-vector); and
F, *H* are *nxn* and *mxn* matrices, respectively.

It is assumed that the process noise u_t and the observation noise v_t are uncorrelated. To start the estimation process, an estimate of the state at $t = 0$ needs to be specified, together with the corresponding error covariance matrix. The estimate of the state at time *t*, using data up to *t*-1 is denoted by $x_{t|t-1}$ and is known as the prior estimate of x_t. The corrsponding error covariance matrix at time *t* is:

$$P_{t|t-1} = E((x_t - x_{t|t-1})(x_t - x_{t|t-1})^T)$$

where *T* denotes transpose. When a new observation z_t becomes available, the estimate $x_{t|t-1}$ is updated according to:

$$x_{t|t} = x_{t|t-1}+K_t(z_t-H_tx_{t|t-1}) \qquad (3)$$

which is the posterior estimate of x_t i.e. the estimate of the state at time t using data up to t. Here K_t is known as the Kalman gain at time t. Note that the term in brackets on the right hand side is the difference between the actual observation and the predicted observation at time t. Thus a large difference between the actual and predicted observations will give a large modification in the state estimate and a small difference results in a correspondingly small modification. The Kalman gain is given by:

$$K_t = P_{t|t-1}H_t^T(H_tP_{t|t-1}H_t^T+R_t)^{-1} \qquad (4)$$

The error covariance matrix $P_{t|t-1}$ is updated by:

$$P_{t|t} = (I-K_tH_t)P_{t|t-1} \qquad (5)$$

Note that $P_{t|t}$ is the error covariance matrix associated with the updated (posterior) estimate of the state at time t.

Finally, new prior estimators of the state and the error matrix at $t + 1$, are obtained by:

$$x_{t+1|t} = F_tx_{t|t} \qquad (6)$$

$$P_{t+1|t} = F_tP_{t|t}F_t^T+Q_t \qquad (7)$$

The Kalman gain at time t+1 can then be calculated and hence the posterior estimate of the state at t+1 and so on.

Equations (3) to (7) comprise the recursive equations for the discrete Kalman Filter. The particular form of the gain K_t given by equation (4) minimises value of trace($P_{t|t}$), i.e the mean square estimation error. The Kalman Filter is therefore the optimal linear estimator for systems with linear observations and dynamics. Note that the gain decreases with increasing observation variance R, and increases with increasing state variance Q. The effect of the observations on the updated state estimate will therefore depend on the relative values of state noise and measurement noise.

The Extended Kalman Filter (EKF)

The estimation procedure described above applies to linear systems. In order to obtain a linear estimator of the form given by Equation (3) for the non-linear system,

$$x_{t+1} = f(x_t,t) + u_t \qquad (8)$$

$$z_t = h(x_t,t) + v_t \qquad (9)$$

some approximations are required. Assuming that the prior estimate of the state at time t, $x_{t|t-1}$, is available, the function $h(x,t)$ is linearised about that estimate giving the Jacobian matrix:

$$H_t = \left[\frac{\partial h}{\partial x}\right]_{x=x_{t|t-1}} \qquad (10)$$

This H_t is then used in Equation (4) for the Kalman gain. The expression for updating the state estimate is:

$$x_{t|t} = x_{t|t-1} + K_t(z_t-h(x_{t|t-1},t)) \qquad (11)$$

The error matrix is updated using Equation (5) with H_t given by Equation (10). The prior estimate of the state at t+1 is obtained by:

$$x_{t+1|t} = f(x_{t|t},t) \qquad (12)$$

and finally, the forward projection of the error matrix via Equation (7) is carried out using the linearisation of $f(x,t)$ about the posterior estimate of x_t at time t, i.e.

$$F_t = \left[\frac{\partial f}{\partial x}\right]_{x=x_{t|t}} \qquad (13)$$

Equations (11) and (12), together with (4), (5) and (7) with F and H given by Equations (10) and (13) are the recursive equations for the Extended Kalman Filter.

The Adaptive Kalman Filter (AKF)

Unknown parameters in the system equations can be treated as system variables to be estimated. Alternatively, Bayesian methodology can be combined with Kalman Filters to obtain a posterior probability distribution of the unknown parameters. Assume that a vector of parameters, A, is unknown. A set of extended Kalman Filters is constructed, one for each value of A in a discrete set $\{A_i : i = 1,\ldots,l\}$. A prior distribution, $p(A_i)$, for A is given, and each time a new observation becomes available, a posterior distribution, $p(A_i|Z_t)$, where Z_t is the set of observations up to and including time t, is updated. This is done as follows:

$$p\,(A_i \mid Z_t) = \frac{p(Z_t \mid A_i)p(A_i)}{p(Z_t)} \qquad (14)$$

where the conditional distribution $p(Z_t|A_i)$, is given by (assuming a scalar output for simplicity, i.e. $m = 1$, and dropping the index on A for convenience of notation),

$$p\,(Z_t \mid A) = \frac{1}{(2\pi)^{1/2}(H_tP_{t|t-1}H_t^T + R_t)^{1/2}} \exp\left(-\frac{(z_t - h(x_{t|t-1},t))^2}{2(H_tP_{t|t-1}H_t^T + R_t)}\right)p(Z_{t-1} \mid A) \qquad (15)$$

where H_t, $x_{t|t-1}$, $P_{t|t-1}$ and R_t may depend on A_i and are obtained by the Extended Kalman Filter method. Note that a 'small' prediction error z_t-$h(x_{t|t-1},t)$, gives a 'high' value of $p(Z_t|A_i)$. Finally, $p(Z_t)$ is calculated by:

$$p\;(Z_t) = \sum_{i=1}^{\ell} p(Z_t \mid A_i)p(A_i)$$

GENERAL MODEL FORMULATION

It is assumed that the population dynamics and observations are governed by the following equations:

$$N_{t+1} = \left(S(N_t - C_t) + (1-S)\left(1 + A\left(1 - \left(\frac{N_t}{K}\right)^z\right)\right)N_t\right)e^{u_t} \qquad (16)$$

$$N_t^{obs} = e_t^v N_t \qquad (17)$$

where N_t is the total population of animals 1 year and older (1+) in year t; C_t is the catch in year t and u_t and v_t are normal random variables with zero mean and variances Q_t and R_t, respectively. This is the well-known Pella-Tomlinson (P-T) model with the parameters: annual survival rate S; pre-exploitation population size (carrying capacity) K; and the resilience parameter A, which is related to $MSYR$ by $MSYR = A(1-S)/S(z/(z+1))$. Note that this is a simplification of the usual P-T models since there is no delay in the dynamics. Note also that the process noise enters by simply multiplying the usual P-T function by a lognormal random variable. This assumption might be questioned, but it enables the dynamics to be written in the required form – as specified in the previous section - by a logarithmic transformation. It should be pointed out that the assumption of normality is not strictly necessary since non-Gaussian assumptions can be accommodated within the extended Kalman Filter.

The state variable is defined to be $x = ln(N)$ and the observation $z = ln(N^{obs})$. The state and observation equations become:

$$x_{t+1} = f(x_t) + u_t$$

$$z_t = x_t + v_t$$

where

$$f(x_t) = \ln\left(S(e^{x_t} - C_t) + (1-S)\left(1 + A\left(1 - \left(\frac{e^{x_t}}{K}\right)^z\right)\right)e^{x_t}\right) \quad (18)$$

The numbers (note that the model is one-dimensional), F_t and H_t used in calculating the Kalman gain and updating the error covariance matrix P (which is simply a scalar variance now) are:

$$F_t = \frac{\partial f}{\partial x}(x) = \frac{S \cdot e^x + (1-S) \cdot e^x \left(1 + A\left(1 - \left(\frac{e^x}{K}\right)^z\right) - A \cdot z \cdot \left(\frac{e^x}{K}\right)^z\right)}{S(e^x - C_t) + (1-S)\left(1 + A\left(1 - \left(\frac{e^x}{K}\right)^z\right)\right)e^x} \quad (19)$$

where the linearisation is about the point $x = x_{t|t}$ and $H_t = 1$.

The possibility of biased observations can be addressed by replacing $z_t = ln(N_t^{obs})$ with $z_t = ln(N_t^{obs})$-ln(B), corresponding to the observation model $N_t^{obs} = BN_t e^{v_t}$, where B is the bias factor.

Adaptive Kalman Filtering can be applied to this model by fixing some of the parameters, i.e. z and S and letting the resilience parameter A (or alternatively, the *MSYR*), the carrying capacity K, and the bias factor B, range over a sequence of discrete values. This gives a three dimensional grid of values (A_i,K_j,B_k) i = 1,2,…,I; j = 1,2,…,J; k = 1,2,…,K, with IJK different sets of these parameter values. To each parameter set there corresponds an extended Kalman Filter and a catch control law is associated with each filter. Whenever a new observation becomes available, the stock estimate, $x_{t|t-1}(A_i,K_j,B_k)$, and the posterior probability distribution, $p(A_i,K_j,B_k|Z_t)$, are updated for each of the IJK parameter sets, (A_i,K_j,B_k) as described in the previous section. Applying a catch control law corresponding to each parameter set (A_i,K_j,B_k), to $x_{t|t}(A_i,K_j,B_k)$, a total of IJK catch limits, $C(x_{t|t}(A_i,K_j,B_k);(A_i,K_j,B_k))$ are obtained, together with the associated posterior probability distribution $p(A_i,K_j,B_k|Z_t)$, i = 1,2,…,I; j = 1,2,…,J; k = 1,2,…,K. Arranging $C(x_{t|t}(A_i,K_j,B_k);(A_i,K_j,B_k))$ in an increasing sequence, the associated probability distribution makes it possible to construct the cumulative distribution function $F(C)$ for the catch limit. Once a percentile γ of this distribution is fixed, the eventual catch limit can be determined by solving:

$$F(C_t) = p(C < C_t) = \gamma \quad (20)$$

for C_t. This percentile will be used as the tuning parameter for the procedure. The procedure is illustrated schematically in Fig. 1.

Specification of the Base Case Model and results

The general model described in the previous section needs to be specified further if it is to be applied to a specific stock. What will be termed the Base Case Model is an application

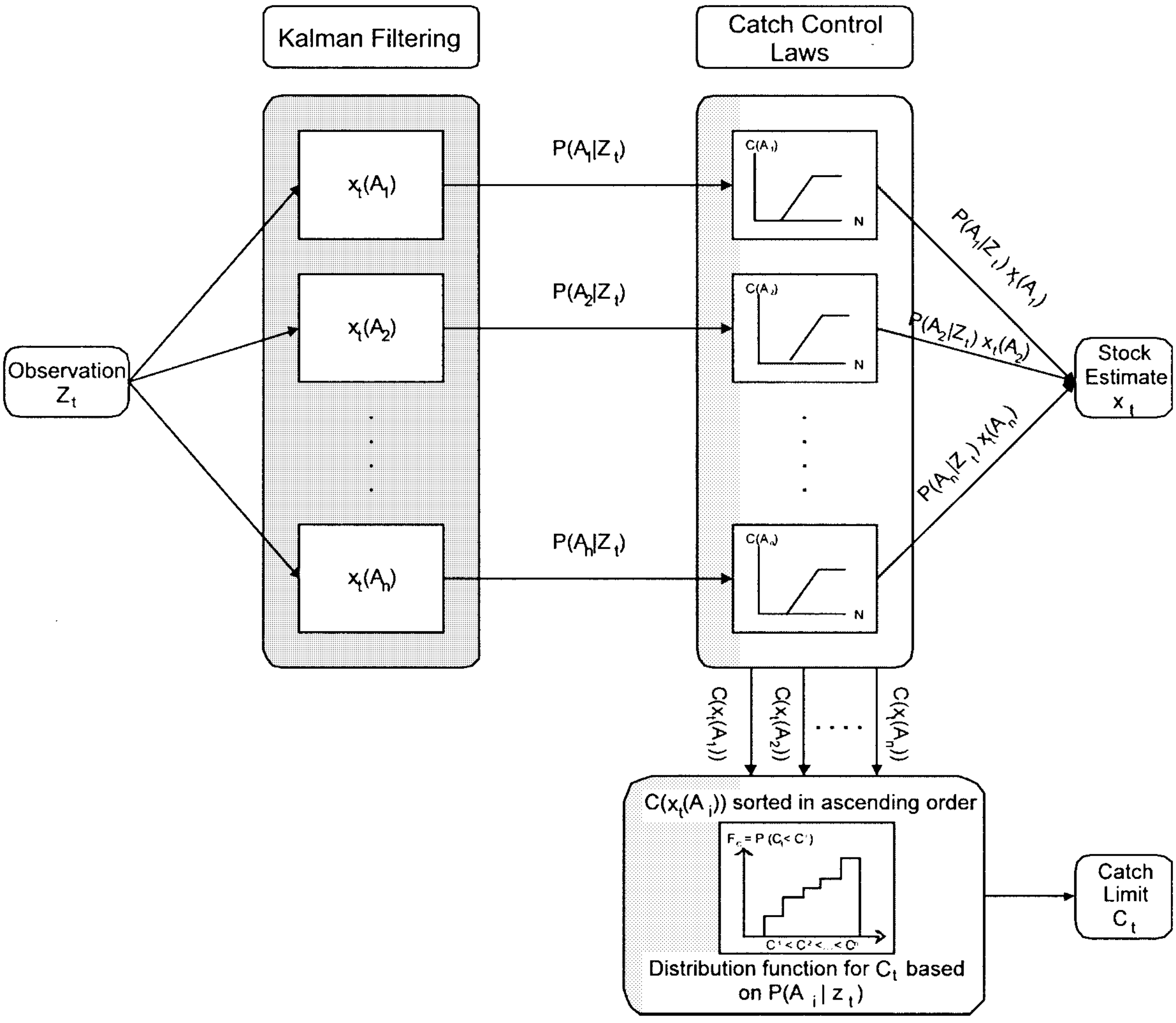

Fig. 1. An overview of the algorithm for setting catch limits.

to the B-C-B stock of bowhead whales and is defined as follows. The two parameters which are kept fixed, S and z, will be set at 0.99 and 2.39 respectively (this value of z corresponds to an *MSY*-level (*MSYL*) of 0.6). The carrying capacity K, ranges from 10,000 to 23,000 in increments of 100 and the values of the resilience parameter A correspond to *MSYR* of 1%, 2%, 3% and 4%. The possibility of bias is not considered in the Base Case Model. There are therefore 131 values of K and four values of *MSYR*, giving a total of 524 filters. It is assumed that the stock is at carrying capacity in 1848 when commercial whaling began. The filters are therefore started in that year, with initial conditions $x_0 = K$ and $P_0 = 0$ (note that there is no initial variance since K is pre-specified in each filter). The state x is projected forward by the equation:

$$x_{t+1} = f(x_t) \tag{21}$$

where $f(x_t)$ is given by Equation (18) and not updated until 1978 when the first observation becomes available. On the other hand, the variance P is projected forward every year by:

$$P_{t+1} = F_t P_t F_t^T + Q_t = (F_t)^2 P_t + Q_t \tag{22}$$

where F_t is given by Equation (19). Note that F_t is a scalar since the model is one-dimensional. The variables x and P are updated by Equations (3) and (5) respectively, whenever a new observation becomes available. There are 10 historical abundance observations between 1978 and 1993, and in the simulation trials, the management procedure will be given abundance observations in 2002, 2004 and then every five years. To each observation there is an associated estimate of the coefficient of variation (*CV*). The variance of the measurement noise v_t, is given by:

$$R_t = Var(v_t) = \ln(1 + CV(N_t^{obs})^2)$$

In order to get an estimate of the variance of the process noise Q, some simulations were carried out using a simplified population model. The survey estimates were generated using a Pella-Tomlinson model without age-structure and without demographic stochasticity, but with noisy observations where the noise was as specified in IWC (2000a). Q was chosen so as to roughly minimise prediction error in a small subset of simulation trials, which gave $Q = 10^{-3}$. Note that a high value of Q will give a high Kalman gain and hence the filter will tend to follow the individual observations, which is not a desirable feature. The sensitivity to the value of Q is investigated in the following section.

There is no prior information on the values of the parameters A and K. The prior distribution for each parameter set (A_i,K_j), i = 1,2,3,4; j = 1,2,…,131, is therefore assumed to be discrete uniform on the specified grid and the first update is in 1978 when the first observation becomes available. Let us first consider the results when the filters are applied to the historical data, i.e. up to 1993 (they will be continued past 1993 in the simulation trials discussed below). Fig. 2 shows the posterior probabilities in 1993, i.e. $p(K|Z_{1993})$, for the four values of *MSYR*. It is clear that although the mode is at 3% *MSYR*, the 1% *MSYR* has the greatest probability mass. This is to be expected since the population trajectory is less sensitive to the initial value, K, when the *MSYR* is low and there is therefore a wider range of K-values which 'agree' with the historical abundance data. Initially all K values are considered to be equally likely. Thus, although the mode of the 1% curve is lower than the 2% and 3% curves, the support of the probability density function is much wider, giving a higher marginal probability when integrating over all K-values. Fig. 3 shows the

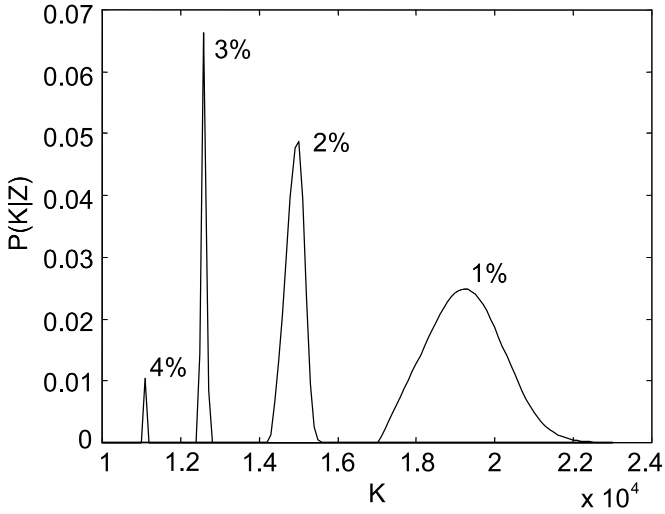

Fig. 2. The posterior probability distribution $p(K|Z_{1993})$ for four values of MSYR.

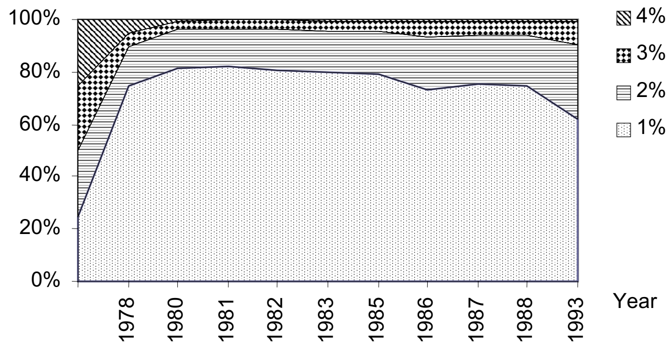

Fig. 3. The evolution of the marginal probability mass function $p(MSYR|Z_t)$ between 1978 and 1993 for four values of MSYR.

evolution of this marginal probability mass function from 1978 to 1993, i.e. *p(MSYR)*. Before 1978 the four *MSY*-rates all have the probability 0.25, but by 1993 *p(MSYR)* are 62%, 28%, 9% and 1% for *MSYR* of 1%, 2%, 3% and 4% respectively. These numbers are in fact the integrals under the four curves in Fig. 2.

The only thing remaining to be specified in the management procedure are the catch control laws corresponding to each filter. The 'H' strike limit rule as defined in IWC (2000a) will be used. This rule gives a strike limit in year t by:

$$H_t = \min\left(Need_t, \begin{cases} 0 & if & N_t < 2000 \\ 0.8RY_t & if & 2000 \le N_t < MSYL \\ 0.9MSY & if & N_t > MSYL \end{cases} \right)$$

where $Need_t$ is the prespecified level of aboriginal need in year t and RY_t is the replacement yield. All parameters refer to the 1+ component of the population (i.e. the total number of animals one year and older). Finally, a maximum of 20% change in strike limits between years is imposed.

An illustration of the AKF-method will now be given by running it on a small subset of the trials defined in IWC (2000b). The main specifications of the trials are given in Table 1, but further details are found in IWC (2000b). The results are given in Table 2. The tuning parameter γ was chosen such that the median final depletion in trial BE01 was approximately 0.78, which is the final depletion when the H-rule with perfect information is applied (in accord with the

Table 1

A subset of trials used to test the AKF method. Initial need is set to 67 in all trials. The Historical survey bias in trials BE09 and BE12 increases linearly in the period 1978-1993 between the values given.

Trial no.	$MSYR_{1+}$	$MSYL_{1+}$	Final need	Historical survey bias	Future survey bias	Survey CV (true, estimated)
BE01	0.025	0.6	201	1	1	0.25, 0.25
BE03	0.025	0.6	201	1	1.5	0.25, 0.25
BE04	0.025	0.6	201	1	0.67	0.25, 0.25
BE06	0.025	0.4	201	1	1	0.25, 0.25
BE07	0.025	0.8	201	1	1	0.25, 0.25
BE09	0.01	0.6	201	0.67→1	1	0.25, 0.25
BE10	0.04	0.8	201	1	1	0.25, 0.25
BE12	0.01	0.6	134	1→1.5	1.5	0.25, 0.10

Table 2

Results of the AKF method for selected trials. The median, the 5^{th} and 95^{th} percentiles are shown for final depletion (D1), lowest depletion (D2), need satisfaction (N9) and the average annual variation (N10).

	D1			D2			N9			N10		
	5^{th}	Median	95^{th}	5^{th}	Median	95^{th}	5^{th}	Median	95^{th}	5^{th}	Median	95^{th}
BE01	0.763	0.777	0.885	0.656	0.711	0.748	0.771	0.985	1.000	0.039	0.051	0.098
BE03	0.791	0.795	0.799	0.656	0.711	0.748	0.970	0.971	0.972	0.039	0.039	0.040
BE04	0.915	0.921	0.929	0.656	0.711	0.748	0.585	0.626	0.657	0.034	0.047	0.064
BE06	0.269	0.451	0.602	0.269	0.407	0.476	0.613	0.702	0.971	0.035	0.058	0.126
BE07	0.931	0.934	0.948	0.717	0.797	0.844	0.965	0.999	1.000	0.039	0.051	0.061
BE09	0.331	0.443	0.593	0.331	0.428	0.509	0.678	0.843	1.000	0.043	0.071	0.134
BE10	0.957	0.967	0.987	0.956	0.960	0.977	0.708	0.973	1.000	0.039	0.056	0.127
BE12	0.210	0.286	0.351	0.210	0.286	0.343	0.776	0.849	0.989	0.045	0.070	0.104

recommendation of the IWC Scientific Committee). Only a few key statistics are given: final depletion (D1), lowest depletion (D2), need satisfaction (N9) and the average annual variation (N10). These statistics are fully defined in IWC (2000a). The results will not be discussed in any great detail here. Suffice it to say that the stock is under-utilised in trial BE04 and over-exploited in trials BE09 and BE12. These results and the possible causes will be discussed in the final section of this paper.

When the AKFs are continued past 1993, the estimated stock trajectories and the further evolution of the posterior probabilities will not only vary between trials but also between each of the 100 simulations comprising each trial. One realisation of trial BE01 is selected here to illustrate the future evolution of *p(MSYR)*. This is shown in Fig. 4. There is a general downward trend in the posterior probability of the 1% *MSYR* and a corresponding upward trend for the 2% and 3% *MSYR* for this particular realisation. Fig. 5 shows the true population trajectory, the estimated trajectory and the actual observations for the same realisation. Note that the estimate of the state x (i.e. the logarithm of the stock size) is the sum of the stock estimates coming from each filter, weighted by the posterior probabilities $p(A_i,K_j|Z_t)$, as illustrated in Fig. 1. The estimated trajectory follows the actual trajectory fairly well, in spite of a wide scattering in the observations.

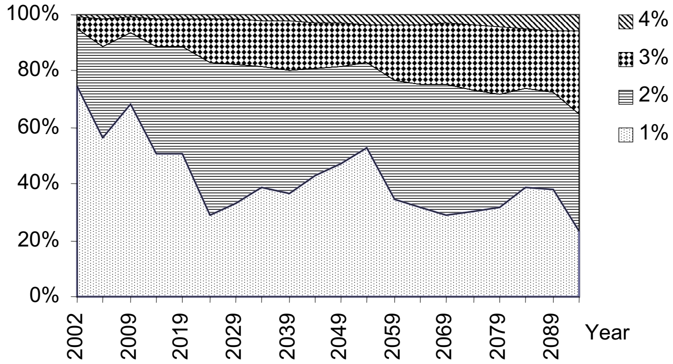

Fig. 4. The evolution of the marginal probabilty mass function $p(MSYR|Z_t)$ over the 100-year management period for one realisation of trial BE01.

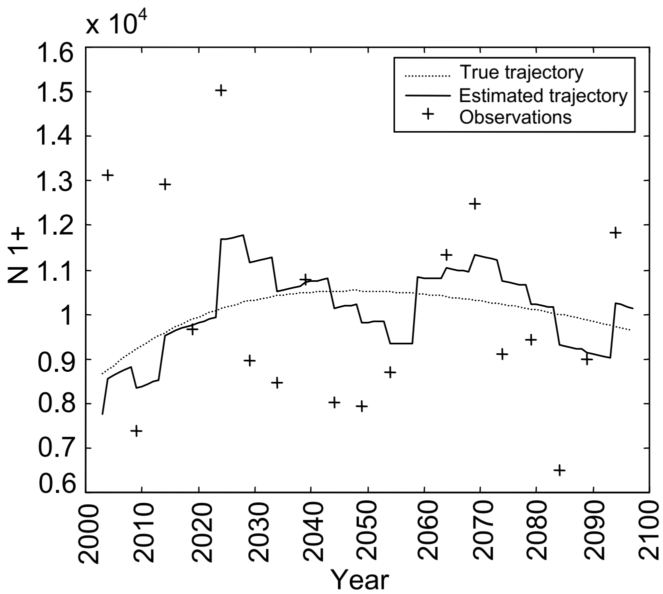

Fig. 5. The true population trajectory, the observations and the estimated trajectory using the Base Case version of the AKF method for one realisation of trial BE01.

SENSITIVITY TESTS

In order to investigate how sensitive the procedure is to the various features which specify the Base Case Model, a few of those specifications were varied.

Firstly, the effect of a finer *MSYR* grid was investigated by letting *MSYR* range from 0.5% to 4% with a step size of 0.5%, giving a total of 8 × 131 = 1,048 filters. Fig. 6 shows the posterior probability distribution $p(K|Z_{1993})$. The mode is now at for 3.5% *MSYR*, but the greatest mass is at 0.5% and 1%. The evolution of the marginal probability mass function $p(MSYR|Z_t)$ between 1978 and 1993 shows more or less the same pattern as the Base Case Model, except for the difference in resolution. This AKF model, was then tested on the same set of simulation trials as the Base Case Model. The value of the tuning parameter γ was chosen so that the median final depletion in trial BE01 was the same as for the Base Case Model. It turned out that the differences were minimal; the median values were more or less unchanged, but the spread was slightly higher in some trials and lower in others. In view of the small differences between the two versions, there appears to be no reason to change the specification of the Base Case Model by taking a finer *MSYR* resolution.

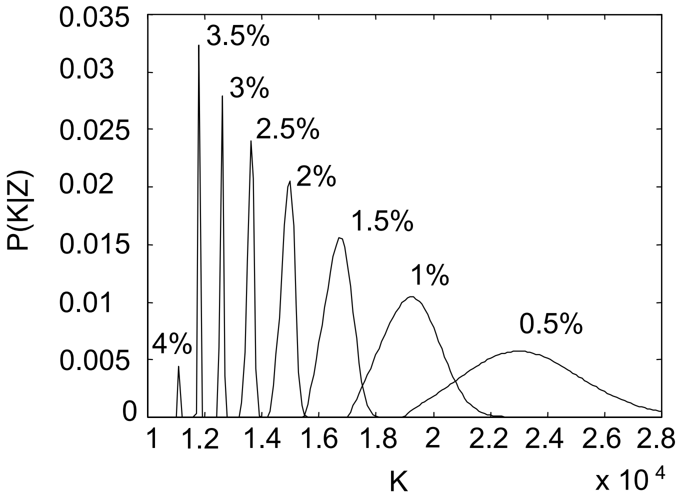

Fig. 6. The posterior probability distributions $p(K|Z_{1993})$, for eight values of MSYR.

Secondly, the effect of varying the process variance Q was examined. The value used in the Base Case Model is 10^{-3}, and here the values, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5} were used. Note that a large value of Q, means that the filter will place a relatively greater weight on the observations and thus the filters tend to follow the observed values more closely, leading to greater fluctuations. This is illustrated in Fig. 7 which shows the same realisation as Fig. 5, but with $Q = 0.01$. Fig. 8 shows the posterior probabilities, $p(MSYR|Z_{1993})$, demonstrating how the $MSYR = 1\%$ filters get a larger share as Q increases. Fig. 9 shows need satisfaction (N9) as a function of Q for trials BE01 and BE09. The results in the former trial are not very sensitive to Q, but a high or a low Q value deplete the stock even further than the Base Case Model in BE09. These results do not provide justification for changing the value of the process variance used in the Base Case Model.

Finally, the possibility of detecting a bias was considered. A three-dimensional grid of filters was used; that is filters with a bias factor of 0.67, 1.0 or 1.5 were included, which gave a total of 4 × 3 × 131 = 1,572 filters. This version of the AKF-method was then applied to trials BE01, BE03 and BE04, which are trials with bias factors of 1.0, 1.5 and 0.67 respectively. A slight modification was made to the trials in that it was assumed that the historical observations were also biased; that is the historical observations were multiplied by

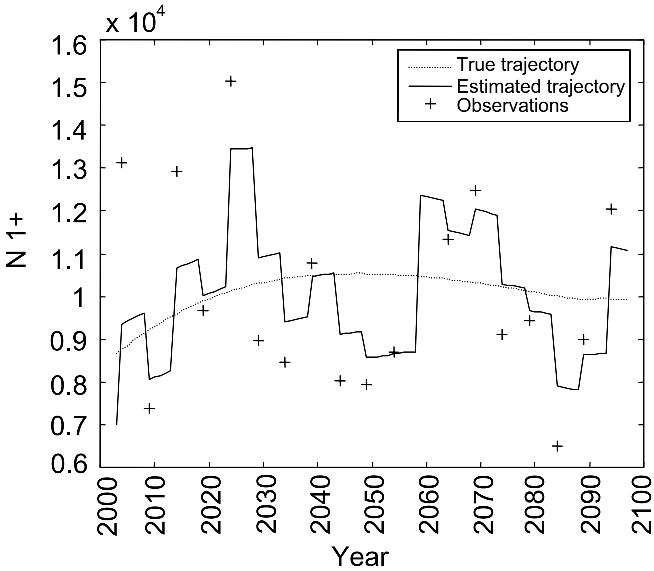

Fig. 7. The true population trajectory, the observations and the estimated trajectory using the AKF method with process variance equal to 0.01 for one realisation of trial BE01.

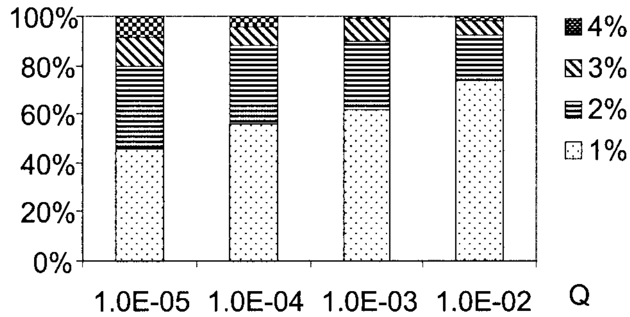

Fig. 8. The marginal probabilty mass function $p(MSYR|Z_{1993})$ for four different values of the process variance Q.

the bias factor relevant to that trial and those numbers were then provided to the management procedure. This was done for reasons of consistency, since it is unlikely that future observations are biased when past ones are not. This change does in fact make the trials 'harder' since the jump in the bias factor helps in identifying a bias case. The results are shown in Fig. 10. Note that need was almost fully satisfied in BE01 and BE03 with the Base Case Model (but with the 5th percentile rather low in the former), but there was considerable under-utilisation in BE04. The results in trial BE01 for the version with bias filters are better than for the Base Case version since the median need is almost the same, but the 5th percentile is much higher; trial BE03 results are almost unchanged, but there is great improvement in BE04 where nearly full need satisfaction is achieved as opposed to only 63% for the Base Case Model. This particular grid of filters only includes cases with a constant bias and the possibility that the bias may be changing through time is not considered. This scenario is addressed in trials BE09 and BE12 where it is assumed that the bias in the historical (1978-1993) observations increased. The AKF with bias filters performs worse than the Base Case Model in trial BE09 where the stock is more heavily depleted (Fig.10c). The marginal posterior probabilities for the 12 combinations of *MSYR* and bias factor after the 100-year management period are shown in Fig. 11 for one realisation of BE01 and BE03. It is clear that those probabilities are highest for the correct bias factor. These results are therefore very

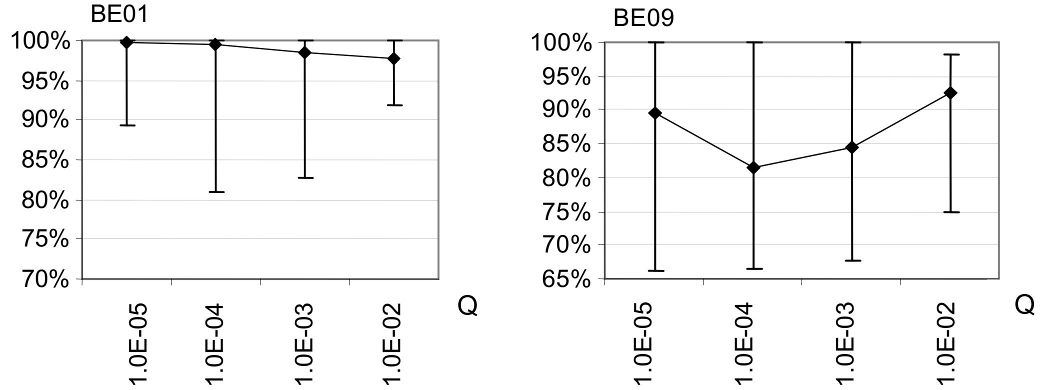

Fig. 9. Comparison of need satisfaction (N9) in trials BE01 and BE09 for four different values of the process variance Q.

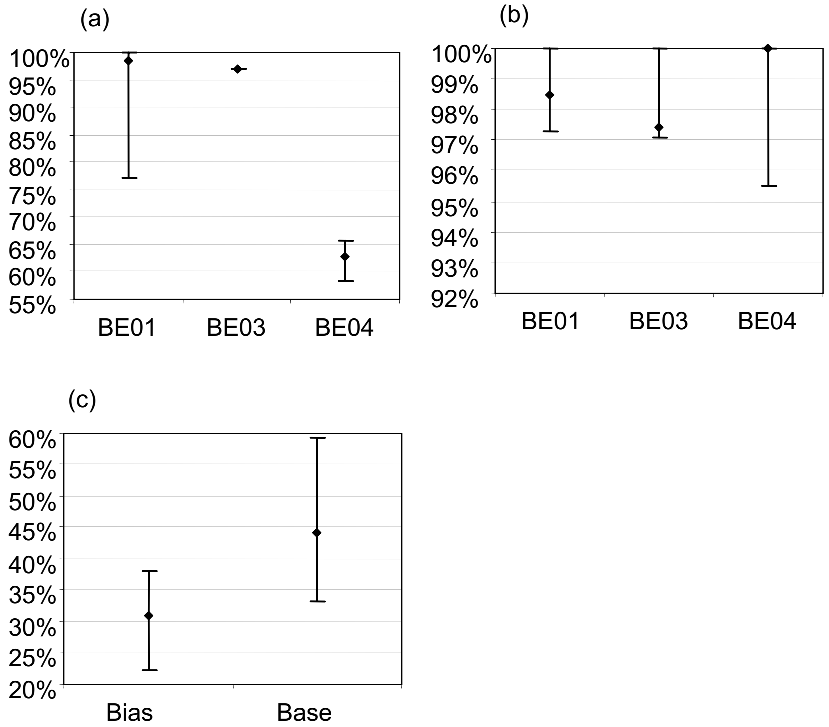

Fig. 10. Need satisfaction (N9) in trials BE01, BE03 and BE04 for the Base Case Model (a) and the version with bias filters (b). Final depletion (D1) in trial BE09 for Base Case Model and the bias filter version (c). Note the different scales on the vertical axis in (a) and (b).

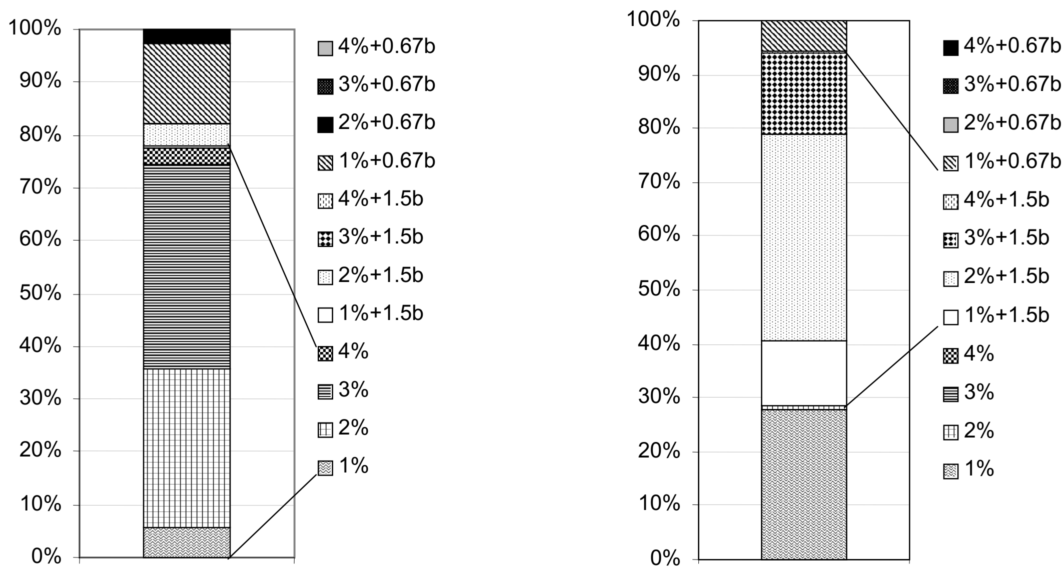

Fig. 11. One realisation of the marginal probabilty mass function $p(MSYR|Z_t)$ after 100 years of management for BE01 (left panel) and BE03 (right panel).

promising and give some cause for optimism that a bias in the data may in fact be correctly identified by the AKF-method.

DISCUSSION

The results of the set of trials presented in Table 2 show adequate or good performance in all except BE04 (low need satisfaction), BE09 and BE12 (too much depletion). Note that the final depletion in BE06 is quite acceptable since the *MSYL* is 0.4. The poor need satisfaction in BE04 is due to the fact that abundance observations are downwards biased by a factor 0.67. The performance in this trial can be greatly improved by including a bias factor in the set of filters (cf. Fig. 10). Trial BE12 is inherently difficult: low *MSYR*, positively biased observations and underestimated CVs. In fact, it may not be possible for a management procedure to perform adequately in this trial and simultaneously attain an acceptable level of need satisfaction in the other trials. Trial BE09 is a low *MSYR* trial, but it should be possible get a reasonable performance by the AKF-method. The 1% *MSYR* has the largest share of the posterior probability $p(MSYR|Z_{1993})$, about 62%. This fraction will generally decrease over the 100-year management period, albeit rather slowly. It would therefore seem that the strike limit is set more or less in accordance with the *MSYR* = 1% case. However, the particular tuning chosen requires a tuning parameter $\gamma = 0.75$. Thus, γ is larger than $p(MSYR = 0.01)$ which means that the eventual strike limit will be somewhat higher than appropriate for the 1% case. It may therefore be worthwhile to explore other tunings.

The sensitivity tests carried out in this paper provide no reasons for changing the Base Case version of the AKF-method, except possibly to add the third dimension to the grid of filters, i.e. to include bias filters. The preliminary results in this direction are promising and this possibility is worth investigating further. However, including filters with constant bias may lead to a deterioration in performance in scenarios where the bias is changing with time as illustrated in Fig. 10c. An obvious way of attempting to address this problem is to add filters with a changing bias and this may be worth exploring further. However, one must be careful not to let the set of filters mimic too closely the trial set. Some separation between the two sets must be maintained. Introducing a filter to correspond to each trial goes against the philosophy behind the process of developing a management procedure. Ideally, the trial details should not be known to the persons developing the management procedure.

The value of the process variance used in the AKF-method is not based on any knowledge of whale population dynamics. Rather, Q is to be regarded as a parameter, which should be set with improved trial performance in mind. It should however, be borne in mind that the Kalman Filter gives less weight to the model and more weight to the observations when Q is large. The filter may therefore disregard the model more or less; it can not distinguish between the different *MSY*-rates for example, and the strike limit is therefore set on the basis of observations alone. This may be the main explanation for the behaviour in Fig. 9. The need satisfaction in BE01 decreases with Q, but increases in BE09. The former is a 2.5% and the latter a 1% *MSYR* trial. The filters are less able to discriminate between 1% and 2.5% as Q increases and the catch levels in the two trials will therefore approach each other.

Some preliminary investigations of the sensitivity of the results to the population model used in the filters have been made using different models, for example a model with delay in the dynamics, (Dereksdóttir and Magnússon, 2000) and a model that finds the K that best fits the historical data for pre-specified values of *MSYR*, thus giving pairs of [*MSYR*,*K*], and thereby a one-dimensional grid of filters. The results of those tests were not dramatically different from the ones presented here, but performance was not improved in general. It should also be noted that the model of the stock dynamics used in the AKF-method is not the same as that used to generate the data in the trials. The method is nevertheless fairly successful in tracking the actual trajectory (an example is given in Fig. 5). Other variations of the AKF-method were also investigated, but this will not be discussed here. Suffices to say, that the model presented here as the Base Case Model gave the best overall performance of all the different versions tried, except possibly for the version of the AKF-method with bias filters. Exploring this version further is a future task. Other tasks for the future, are to explore whether performance can be improved by amending the catch control laws and to look at the sensitivity of the results to the values of the fixed parameters z and S.

ACKNOWLEDGEMENTS

We wish to thank Cherry Allison for her assistance in setting up and running the Evaluation trials. We also thank the IWC for a contribution from the Developers' Fund and two reviewers for their comments.

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